CMA INTER

P9A: OPERATIONS MANAGEMENT (SOLUTIONS)

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2. OPERATIONS PLANNING

Answer for Q.NO.1.

Computation of Trend Values

Years	Time Deviation from 2004 X	Sales in (Rs. 000 units) Y	Squares of time dev. X^2	Product of time deviations and sales XY
2015	–3	80	9	–240
2016	–2	90	4	–180
2017	–1	92	1	–92
2018	0	83	0	0
2019	+1	94	1	+94
2020	+2	99	4	+198
2021	+3	92	9	+276

n = 7 $\Sigma X = 0$ $\Sigma Y = 630$ $\Sigma X^2 = 28$ $\Sigma XY = + 56$

Regression equation of Y on X

Y = a + bX

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{56}{28} = 2$$

Hence regression equation comes to Y = 90 + 2X. With the help of this equation we can project the trend values for the next three years, i.e. 2022, 2023 and 2024.

Y_{2008} = 90 + 2(4) = 90 + 8 = 98 (000) units.

Y_{2009} = 90 + 2(5) = 90 + 10 = 100 (000) units.

Y_{2010} = 90 + 2(6) = 90 + 12 = 102 (000) units.

Answer for Q.NO.2.

Computation of trend values of sales

Year	Time deviations from the middle of 2004 and 2005 assuming 6 months = 1 unit	Sales (in lakh Rs.)	Squares of time deviation	Product of time deviation and sales
	X	Y	X2	XY
2016	-5	100	25	-500

2017	-3	110	9	-330
2018	-1	115	1	-115
2019	+1	120	1	+120
2020	+3	135	9	+405
2021	+ 5	140	25	+700
n = 6	ΣX = 0	ΣY = 720	ΣX^2 = 70	ΣXY = 280

Regression equation of Y on X:

Y = a + bX

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{720}{6} = 120$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{280}{70} = 4$$

Hence regression equation comes to Y = 120 + 4X

Sales forecast for the next years, i.e., 2022-26

Y_{2022} = 120 + 4 (+7) = 120 + 28 = Rs. 148 lakhs

Y_{2023} = 120 + 4 (+9) = 120 + 36 = Rs. 156 lakhs

Y_{2024} = 120 + 4 (+11) = 120 + 44 = Rs. 164 lakhs.

Y_{2025} = 120 + 4 (+13) = 120 + 52 = Rs. 172 lakhs.

Y_{2026} = 120 + 4 (+15) = 120 + 60 = Rs. 180 lakhs.

Answer for Q.NO.3.

Computation of trend values

Population (in lakhs)	**Sales of CTV (in thousands)**	**Squares of the population**	**Product of population and sales of colour TV**
X	Y	X^2	XY
5	9	25	45
7	13	49	91
8	11	64	88
11	15	121	165
14	19	196	266
ΣX = 45	Σy = 67	ΣX^2 = 455	ΣXY = 655

Regression equation of Y on X

Y = a + bX

To find the values of a and b, the following two equations are to be solved

ΣY = na + bΣX (i)

ΣXY = aΣX + bΣX2 (ii)

By putting the values we get

67 = 5a + 45b (iii)

655 = 45a + 455b (iv)

Multiplying equation (iii) by 9 and putting it as no. (v) we get, 603 = 45a + 405b ... (v)

By deducting equation (v) from equation (iv); we get 52 = 50b

$$b = \frac{52}{60} = 1.04$$

By putting the value of b in equation (iii), we get

67 = 5a + 45 × 1.04

or, 67 = 5a + 46.80

or, 67-46.80 = 5a

or, 5a = 20.20

or, a = $\frac{20.20}{5}$

or a = 4.04

Now by putting the values of a and b the required regression equation of Y on X, is

$$Y = a + bX \text{ or, } Y = 4.04 + 1.04X$$

When X = 10 lakhs than

Y = 4.04 + 1.04 (10)

or, Y = 4.04 + 10.40 or 14.44 thousand CTV sets.

Similarly for town having population of 20 lakhs, by putting the value of X = 20 lakhs in regression equation

Y = 4.04 + 1.04 (20)

= 4.04 + 20.80 = 24.84 thousands CTV sets.

Hence expected demand for CTV for two towns will be 14.44 thousand and 24.84 thousand CTV sets.

Answer for Q.NO.4.

Computation of trend value

Population (in lakhs) X	**No. of scooters demanded Y**	**Squares of population X^2**	**Product of population and No. of scooters demanded XY**
4	4,400	16	17,600
6	6,600	36	39,600
7	5,700	49	39,900
10	8,000	100	80,000
13	10,300	169	1,33,900
$\Sigma X = 40$	$\Sigma Y = 35,000$	$\Sigma X^2 = 370$	$\Sigma XY = 3,11,000$

Regression equation of Y on X

Y = a + bX

To find the values of a and b we will have to solve the following two equations

$\Sigma Y = na + b\Sigma X$ ... (i)

$\Sigma XY = a\Sigma X + b\Sigma X2$ .... (ii)

By putting the values, we get

35,000 = 5a + 40b ... (iii)

3,11,000 = 40a + 370b ... (iv)

By multiplying equation no. (iii) by 8 putting as equation (v) we get,

2,80,000 = 40a + 320b ... (v)

By subtracting equation (v) from equation (iv), we get

31,000 = 50b

or, 50b = 31,000

or, $b = \frac{310}{50} = 620$

By substituting the value of b in equation no. (iii), we get

35,000 = 5a + 40b

Or 35,000 = 5a + 40 × 620

or 35,000 = 5a + 24,800

or 10,200 = 5a

or $a = \frac{10200}{5} = 2040$

Now putting the values of a and b the required regression equation of Y on X, is

Y = a + bX or, Y = 2040 + 620 X

When X = 16 lakhs then Y = 2040 + 620 (16)

or Y = 2040 + 9920

or Y = 11,960

Hence, the expected demand of scooters for a town with a population of 16 lakhs will be 11,960 scooters.

<u>Answer for Q.NO.5.</u>

Step 1: Calculate the processing time needed in hours to produce product x, y and z in the quantities demanded using the standard time data.

Product	Annual demand (units)	Standard processing time per unit (Hrs.)	Processing time needed (Hrs.)
X	300	4.0	300 x 4 = 1200 Hrs.
Y	400	6.0	400 x 6 = 2400 Hrs.

Product	Annual demand (units)	Standard processing time per unit (Hrs.)	Processing time needed (Hrs.)
Z	500	3.0	500 × 3 = 1500 Hrs.
			Total = 5100 Hrs

Step 2 : Annual production capacity of one machine in standard hours = 8 × 250 = 2000 hours per year

Step 3 : Number of machines required

$$= \frac{\text{Work load per year}}{\text{Production capacity per machine}} = \frac{5100}{2000} = 2.55 \text{ machines} = 3 \text{ machines}$$

Answer for Q.NO.6.

Actual output

$$\text{Efficiency of the plant} = \frac{\text{Actual output}}{\text{Effective Capacity}} = \left(\frac{36000}{40000}\right) \times 100 = 90$$

$$\text{Utilisation} = \left(\frac{\text{Actual output}}{\text{Design Capacity}}\right) = \left(\frac{36000}{50000}\right) \times 100 = 72\%$$

Answer for Q.NO.7.

(i) The bottle neck centre is the work centre having the minimum capacity. Hence, work centre 'C' is the bottleneck centre.

(ii) System capacity is the maximum units that are possible to produce in the system as a whole. Hence, system capacity is the capacity of the bottle neck centre i.e., 340 units.

(iii) System efficiency = $\frac{\text{Actual output}}{\text{System Capacity}}$

$$= \frac{300}{340} \times 100 \text{ (i.e., maximum possible output)} = 88.23\%$$

Answer for Q.NO.8.

(i) Break-even point

Let Q be the break even point.

FC = Fixed cost, R = Revenue per unit, VC = Variable cost

At, BEP, TR = FC + TVC

or, Revenue p.u × Q = FC + VCp.u. × Q

Q (R - VC) = FC

$$Q = \frac{FC}{R - VC}$$

Let Q1 be the break-even-point for one machine option

$$\text{Then, } Q1 = \frac{1200}{(50-20)} = \frac{1200}{30} = 400 \text{ units}$$

(Not within the range of 0 to 300)

Let Q2 be the break-even-point for two machines option.

$$\text{Then } Q2 = \frac{1500}{(50-20)} = \frac{1500}{30} = 500\text{units}$$

(within the range of 301 to 600)

Let Q3 be the break-even-point for three machines option.

$$\text{Then, } Q1 = \frac{21000}{(50-20)} = \frac{21000}{30} = 700\text{units}$$

(with in the range of 601 to 900)

(ii) The projected demand is between 600 to 650 units.

The break even point for single machine option (i.e., 400 units) is not feasible because it exceeds the range of volume that can be produced with one machine (i.e., 0 to 300).

Also, the break even point for 3 machines is 700 units which is more than the upper limit of projected demand of 600 to 650 units and hence not feasible. For 2 machines option the break even volume is 500 units and volume range is 301 to 600.

Hence, the demand of 600 can be met with 2 machines and profit is earned because the production volume of 600 is more than the break even volume of 500. If the manager wants to produce 650 units with 3 machines, there will be loss because the break even volume with three machines is 700 units. Hence, the manager would choose two machines and produce 600 units.

Answer for Q.NO.9.

This situation can be solved using Factor Rating Method. The steps are:

In the first stage the expert team needs to give weightage to the factors. This can be done in many ways. In the following one simple way is explained.

A possible approach:

Suppose, the experts rate each factor on a scale 1 to 5 (1: least important and 5: Most important)

Factor	**Rating**						
	E-1	E-2	E-3	E-4	E-5	**Row**	**Weight**
F1	4	3	4	4	3	18	18/68
F2	5	5	5	5	4	24	24/68
F3	3	4	4	3	5	19	19/68
F4	2	1	2	1	1	7	7/68
						68	

There may be other ways (e.g., AHP method). Let us now come back to our problem. Let us assume the factors are following weights.

Factors	**Weight**
F1	0.3
F2	0.2

F3	0.1
F4	0.4
Total	1.0

The experts are requested to rate each of the location alternatives with respect to the factors, e.g., 10: Most beneficial and 1: Least beneficial

Factors	**Alternatives**		
	L1	L2	L3
F1	10	9	7
F2	7	3	10
F3	7	5	10
F4	6	8	5

So the complete table becomes

Factors	**Weight**	**Alternatives**		
		L1	L2	L3
F1	0.3	10	9	7
F2	0.2	7	3	10
F3	0.1	7	5	10
F4	0.4	6	8	5
	Best Location	**7.5**	7	7.1

Example of calculation

for L1 : 0.3 × 10 + 0.2 × 7 + 0.4 × 6 = 3 + 1.4 + 0.7 + 2.4 = 7.5

As per the weighted score Location L1 is the best location

<u>Answer for Q.NO.10.</u> .

Loc	**V_1**	**x_i**	**y_i**	**$v_i\ x_i$**	**$v_i\ y_i$**
L1	200	30	100	6000	20000
L2	100	90	120	9000	12000
L3	100	130	130	13000	13000
L4	200	60	40	12000	8000
600		Total	40,000	53,000	

Therefore, $\sum V_i = 600$; $\sum v_i x_i = 40000$

$\sum v_i y_i = 53000$

COG location is given by (X, Y)

$$X = \frac{\sum V_i X_i}{\sum V_i}$$

= 40000/600 = 200/3

$$X = \frac{\sum V_i Y_i}{\sum V_i} = 53000 / 600\ 265 / 3$$

Answer for Q.NO.11.

The distance matrix of the present layout :

From / To	A	B	C	D	E	F
A		1	1	2	2	3
B			2	1	3	2
C				1	1	2
D					2	1
E						1
F						–

(ii) Computation of total cost matrix (combining the inter departmental material handling frequencies and distance matrix.

From / To	A	B	C	D	E	F	Total
A		0	90	320	100	0	510
B			140	0	300	260	700
C				20	0	0	20
D					360	10	370
E						40	40
F							–
Total							1,640

If the departments are interchanged, the layout will be represented as shown below.

A	F	E
B	D	C

The distance matrix and the cost matrix are represented as shown.

From / To	A	B	C	D	E	F
A		1	3	2	2	1
B			2	1	3	2
C				1	1	2
D					2	1
E						1
F						

Total cost matrix for the modified layout.

From / To	A	B	C	D	E	F	Total
A	–	0	270	320	100	0	690
B			140	0	300	260	700

C				20	0	0	20
D					360	10	370
E						40	40
F							–
Total							1,820

The interchange of departments C and F increases the total material handling cost. Thus, it is not a desirable modification.

<u>Answer for Q.NO.12.</u>

Keep the departments E and F at the current locations. From the Trip Matrix, C is having maximum no. of trips from E&F. So C must be as close as possible to both E and F, put C between them. Place A directly south of E, and B next to A. All of the heavy traffic concerns have been accommodated. Department D is located in the remaining place.

The proposed layout is shown in figure below. The load distance (ld) scores for the existing and proposed layout are shown below. As ld score for proposed layout is less, the proposed layout indicates improvement over existing.

E	C	F
A	B	D

Comparative Analysis : Current and Proposed Layout :-

Dept. Pair	No. of Trips (1)	Existing plan		Proposed plan	
		Distance (3)	Load × Distance (1 × 3)	Distance (2)	Load × Distance (1 × 2)
A–B	8	2	16	1	8
A–C	3	1	3	2	6
A–E	9	1	9	1	9
A–F	5	3	15	3	15
B–D	3	2	6	1	3
C– E	8	2	16	1	8
C–F	9	2	18	1	9
D–F	3	1	3	1	3
E–F	3	2	6	2	6
Total			**92**		**67**

As 'ld' score of the proposed layout is lower than the existing one, there is an improvement in the new layout.

Answer for Q.NO.13.

We notice quite obviously that from Di to Di (i = 1, 2….6), there is no movement.

From D2 to D1, the average movement is 10 (circle) and from D1 to D2 the average movement is 8 (circle)

Therefore, the combined average traffic movement from D1 to D2 is = (10 + 8) = 18

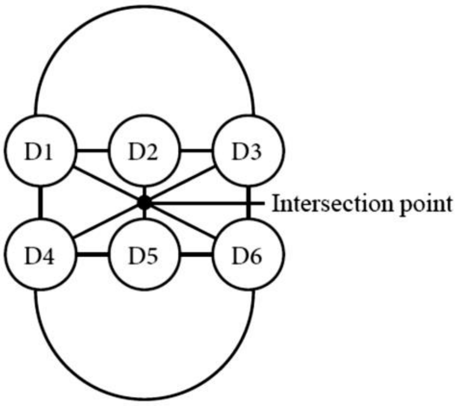
Let us now take another pair, e.g., D4 and D2

Movement	Avg traffic
D4 → D2	10 (red circle)
D2 → D4	25 (Green circle)

Therefore, the combined average traffic movement is 35. Proceeding in the same way, we get the combined average traffic movement for all pairs as follows:

	D1	D2	D3	D4	D5	D6
D1	–	18	30	0	20	6
D2		–	12	35	10	8
D3			–	25	8	8
D4				–	30	7
D5					–	10
D6						–

Let us now draw the initial layout again.



Adjacent Pairs	Non-adjacent Pairs
D1 & D2	D1 & D3
D2 & D3	D1 & D6
D3 & D6	D3 & D4
D6 & D5	D4 & D6
D5 & D4	
D2 & D5	
D1 & D4	
D1 & D5	

D3 & D5 D2 & D4 D2 & D6	

Let us now concentrate on the non-adjacent pairs

Non-adjacent Pair	**Distance**
D1 & D3	(D1 → D2; D2 → D3) D1 → D3 : 2 nodal points Hence, distance is 2
D1 & D6	D1 → D6 = D1 → P & P → D6 Distance = 2
D3 & D4	D3 → D4 = D3 → P & P → D4 Distance = 2
D4 & D6	D4 → D6 = D4 → D5 & D5 → D6 Distance = 2

The combined average traffic movement between any two non-adjacent nodes is called the load distance. Our objective is to reduce the load distance.

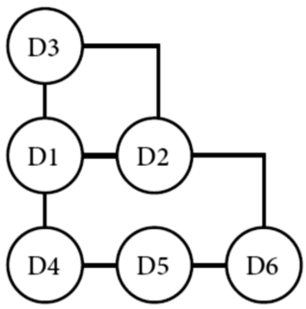
Non-adjacent Pair	**Distance**
D1 & D3 D3 & D4 D1 & D6 D4 & D6	30 × 2 = 60 25 × 2 = 50 6 × 2 = 12 7 × 2 = 14
	Total = 136

Note that for getting the load values, please refer table (Solution).

To meet our objective, we find the highest load distance, i.e., 60. Therefore, we need to rearrange the nodes.

We notice that from D1 to D3 and back, the highest traffic is involved. Therefore, we need to rearrange their positions to make them adjacent as follows:

First rearrangement

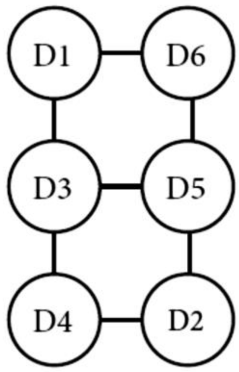


The revised non-adjacent pairs and load distance calculation is given below

Non-adjacent Pair	distance	Load distance
D4 & D6	2	14
D1 & D6	2	12
D3 & D6	2	16
D3 & D5	2	16
D3 & D4	2	50
		108

We notice that there is an improvement. However, now the pair of D3 and D4 creates the problem. Therefore, we need to make them adjacent through rearrangement as follows:

2nd Arrangement



The revised non-adjacent pairs and load distance (after second arrangement) is given below

Non-adjacent Pair	Load	distance	load-distance
D_1D_4	2	0	0
D_6D_2	2	8	16
D_1D_2	2	18	36
D_6D_4	2	7	14
			66

Through trial and error approach we arrive at a considerable improvement. Therefore, the above layout (2nd Arrangement) is the acceptable one.

Answer for Q.NO.14.

Different pure strategies are

Plan I In this pure strategy, the actual demand is met by varying the work force size. This means that during the period of low demand, the company must fire the workers and during the period of high demand the company must hire workers. These two steps involve associated costs. In this strategy, the production units will be equal to the demand and values in each period. The cost of the plan is computed in the table below,

Quarter	Demand	Cost of increasing Production level (Rs.)	Cost of decreasing Production level (Rs.)	Total cost of plan (Rs.)
1	270	—	—	—
2	220	—	50 × 200 = 10,000	10,000
3	470	250 × 150 = 37,500	—	37,500
4	670	200 × 150 = 30,000	—	30,000
5	450	—	220 × 200 = 44,000	44,000
6	270	—	180 × 200 = 36,000	36,000
7	200	—	70 × 200 = 14,000	14,000
8	370	170 × 150 = 25,500	—	25,500
	Total			1,97,000

Plan II In this plan, the company computes the average demand and sets its production capacity to this average demand. This results in excess of units in some periods and also shortage of units during some other periods. The excess units will be carried as inventory for future use and shortage of units can be fulfilled using future inventory.

The cost of the plan II is computed in the table below. The plan incurs a maximum shortage of 255 units during quarter 5. The firm might decide to carry 255 units from the beginning of period 1 to avoid shortage. The total cost of the plan is Rs. 96,500.

Quarter	Demand forecast	Cumulative demand	Production level = Av. Demand = 2920 ÷ 8	Cumu. prod. level	Inventory = (Cum. Production – Cum. Demand)	Adjusted inventory with 255 at beginning of period 1	Cost of holding inventory (Rs.)
1	270	270	365	365	95	350	17,500
2	220	490	365	730	240	495	24,750
3	470	960	365	1095	135	390	19,500
4	670	1630	365	1460	–170	85	4,250
5	450	2080	365	1825	–255	0	0
6	270	2350	365	2190	–160	95	4,750
7	200	2550	365	2555	5	260	13,000
8	370	2920	365	2920	0	255	12,750
	Total						96,500

Plan III

Normal Production Capacity is assumed to be 200 units i.e. Minimum of the demand values. The additional demand other than the normal capacity is met by subcontracting. The cost of the plan III amounts to Rs. 1,32,000 as shown in table below.

Quarter	Demand forecast	Production units	Subcontract units	Incremental cost @ Rs. 100/units
1	270	200	70	70 × 100 = 7,000
2	220	200	20	20 × 100 = 2,000
3	470	200	270	270 × 100 = 27,000
4	670	200	470	470 × 100 = 47,000
5	450	200	250	250 × 100 = 25,000
6	270	200	70	70 × 100 = 7,000
7	200	200	0	0
8	370	200	170	170 × 100 = 17,000
			Total	= 1,32,000

The total cost of pure strategies is given below. On observation Plan II (Changing inventory levels) has the least cost.

Plan	Total cost (Rs.)
Plan I	1,97,000
Plan II	96,500
Plan III	1,32,000

<u>Answer for Q.NO.15.</u>

Chart of Production Requirement

Month	Forecasted Demand	Production Days	Demand/Day	Cumulative Production Days	Cumulative Demand
Jan	220	22	10	22	220
Feb	90	18	5	40	310
Mar	210	21	10	61	520
Apr	396	22	18	83	916
May	616	22	28	105	1532
Jun	700	20	35	125	2232
Jul	378	21	18	146	2610
Aug	220	22	10	168	2830
Sep	200	20	10	188	3030
Oct	115	23	5	211	3145

Nov	95	19	5	230	3240
Dec	260	20	13	250	3500
Total	3500				

(a) Average Requirement = Total Demand / Total Production Days = 3500/25 = 14units/day

(b) Inventory Balance = ∑Production – ∑Demand

Showing the ending Inventory Balance and Ending Balance with Negative Shortage.

Month	Production at 14/day	Forecasted Demand	Inventory Change	Ending Inventory	Balance Ending Balance adjusted in the month of Jan
Jan	308	220	88	88	654
Feb	252	90	162	250	816
Mar	294	210	84	334	900
Apr	308	396	-88	246	812
May	308	616	-308	-62	504
Jun	280	700	-420	-482	84
Jul	294	378	-84	-566	0
Aug	308	220	88	-478	88
Sep	280	200	80	-398	168
Oct	322	115	207	-191	375
Nov	266	95	171	-20	546
Dec	280	260	20	0	566

(c) Maximum Inventory required in storage = 900 units (in the above table Column 6)

Average Inventory Balance = 460 units

Solution to Plan 1 (Varying Inventory):

Inventory Cost = Carrying Cost + Storage Cost

Carrying Cost = 0.20 × 460×100 = 9200

Storage Cost = 900 × 0.90 = 810

Inventory Cost = Rs.10010

Solution to Plan 2 (Varying Employment):

Rs.12000 (Given)

Comparing Plan 1 and Plan 2 we see that Plan 1 is lower.

In case of Plan 3:

it is given that Produce at 10 units per day, vary inventory and sub-contract.

A production rate of 10 units per day exceeds demand only 3 months (Feb, Oct, Nov)

Month	Production at 10/day	Forecasted Demand	Inventory Change
Jan	220	220	0

Feb	**180**	**90**	**90**
Mar	210	210	0
Apr	220	396	-176
May	220	616	-396
Jun	200	700	-500
Jul	210	378	-168
Aug	220	220	0
Sep	200	200	0
Oct	**230**	**115**	**115**
Nov	**190**	**95**	**95**
Dec	200	260	-60

The Inventory Accumulated During these Years must be carried at a cost of (20%) (Rs.100) /12 Months = Rs.1.67 per unit month. Units are Carried until they can be used to help meet demand in a subsequent month

Assume, an equilibrium condition where the excess production from OCT and NOV (150 Units) is on hand JAN

Month	Demand	Production at 10/day	Inventory to carry	Inventory carried until	No. of Months	Cost at $1.67 per unit month
Initial			150	150 units to April	3	750
Feb	90	180	90	26 units to April	2	87
				64 units to May	3	320
Oct	115	230	115	60 units to Dec	2	200
				55 units to Year End	3	275
Nov	95	190	95	95 units to Year End	2	317
					Total	1952

Therefore, Inventory Cost from above = Rs.1952

Calculating Marginal Cost of Sub-contracting:

The marginal cost of sub-contracting

Number of units = Demand – Production = 3500 – (10 × 250) = 1000 units for sub-contracting

Therefore, Cost per unit = Rs.107 – Rs.100 = Rs.7 per unit

Therefore, Marginal Cost = 1000 units × Rs.7 per unit = Rs.7000

The total Cost of Plan 3 = Inventory Cost + Sub-contracting cost = 1952 + 7000 = Rs.8952

Plan 4:

This plan differs from plan 3 only in the marginal cost which is now due to overtime rather than sub-contracting.

So, Inventory cost (same as plan 3) i.e., Rs.1952 and Marginal cost of Overtime = 1000 units × rate of Rs.10 per unit = Rs.10,000

Therefore, total cost of Plan 4 = Rs.10,000 + Rs.1952 = Rs.11952

Table: Comparison of Plans

Plan	Strategy	Cost
Plan 1	Pure Strategy (Vary Inventory)	Rs.10010
Plan 2	Pure Strategy (Vary Employment)	Rs.12000
Plan 3	Mixed Strategy (Sub-contract and Vary Inventory)	Rs.8952
Plan 4	Mixed Strategy (Overtime and Vary Inventory)	Rs.11952

<u>Answer for Q.NO.16.</u>

We are given that,

A = Annual demand = 3,000 × 12 = 36,000 units per annum ; S = Ordering Cost = Rs. 25;

C = Inventory carrying cost = 2 + 20% of Rs. 20 = 2 + 4 = Rs. 6

(i) EOQ = $\sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{6}} = \sqrt{3,00,000} = 548 \text{units(approx.)}$

Total cost = Ordering Cost + Cost of purchasing the material + Storage cost

= (36,000 / 548) × 25 + (36,000 × 20) + (548/2) × 6 [∵ Storage cost = Average Inventory × Inventory carrying cost

= Rs. 1642.33 + 7,20,000+ 1,644 = Rs. 7,23,286. = $= \frac{EOQ}{2} \times 6$]

(ii) When the company has an option to order between 3000 and 6000 units, the EOQ should be calculated with a reduction in price by 5% (due to concession); The purchase price = 95% of Rs. 20 = Rs. 19.

A = 36,000 units per annum; S = Rs. 25; C = 2 + 20% of 19 = 2 + 3.80 = Rs. 5.80

EOQ = $\sqrt{\frac{2 \times 36000 \times 25}{5.80}} = \sqrt{\frac{1,80,000}{5.80}} = 557 \text{units(approx.)}$

Total cost = (36,000/557) × 25 + (36,000 × 19) + (557/2) × 5.80

= Rs. (1,615.79 + 6,84,000 + 1,615.30) = Rs. 6,87,231.09

For monthly order quantity being 3000 units or more but less than 6000 units

EOQ = 557 units

No. of orders per year = $\frac{\text{Yearly demand}}{\text{EOQ}} = \frac{36000}{557} = N(\text{Let})$

No. of orders per month = $\frac{N}{12} = \frac{36000/557}{12} = 5.385 = 6(\text{Say}) = N*$

Quantity to be orderd per month = N* × EOQ = 6 ×557 = 3342 units

This quantity lies in the range of 3000 to 6000 units

Hence the EOQ (557 units) can be considered to be a feasible quantity for availing 5% discount on Purchase Price.

(iii) When the company orders more than 6,000 units purchase price = 90% of Rs. 20 (because 10% concession)

= Rs. 18; A = 36,000 units per annum; S = Rs. 25; C = 2 + 20% of Rs. 18

= 2 + 3.60 = 5.60

EOQ = $= \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{5.60}}$ = 567 units app.

For monthly order quantity more than or equal to 6000 units

EOQ = 567 units

No of orders per month = $\frac{36000/567}{12} = 5.29 = 6(\text{Say}) = N*$

Qty. to be ordered per month = N* × EOQ = 6×567 = 3402 units

This quantity does not lie in the range of 6000 or more units.

Hence the EOQ (567 units) can not be considered as feasible quantity for availing 10% discount on Purchase Price.

To understand the effect of 10% on Total Cost, we consider the minimum value of price break quantity of this range i.e. 6000 units to be the optimum order quantity and calculate.

Total Cost as follows —

TC = Ordering Cost + Cost of Purchasing the material + Storage Cost

= $\frac{36000}{6000} \times 25 + 36000 \times 18 + \frac{6000}{2} \times 5.60$

= 150 + 648000 + 16800 = Rs. 6,64,950

Hence the total cost will be minimum (Rs. 6,64,950) if orders are placed in lot size of 6000 units.

<u>Answer for Q.NO.17.</u>

(1) Economic Order Quantity:

Annual usage of tubes (A) = Normal usage per week × 52 weeks

= 100 tubes × 52 weeks

= 5,200 tubes.

Ordering cost per order (S) = Rs. 100.

Inventory carrying cost per unit per annum (C) = 20% of Rs. 500 = Rs. 100.

$$EOQ=\sqrt{\frac{2AS}{C}}=\sqrt{\frac{2\,x\,200units\ x\ \bullet 100}{\bullet 100}}$$ = 102 units (approx.).

(A) Evaluation of order size of 1,500 units at 5% discount

No. of orders $=\frac{5{,}200units}{1{,}500units}$ = 3.46 or 4 (in case of a fraction, the next whole number is considered).

	Rs.
Ordering cost (No. of order per year at Rs. 100 per order)	400
Carrying cost of average inventory:	
$\frac{1{,}500units}{2}\,x\,Rs.(50 less 5\%)\,x\,\frac{20}{100}$	71,250
Total annual cost (excluding item cost)	71,650
(B) Annual cost if EOQ (102 units) is adopted :	Rs.
Ordering cost: 5,200 ÷ 102 or 51 orders per year at Rs.100 per order	5,100
Carrying cost of average inventory $\frac{102units}{2}\,x\,Rs.500\,x\,\frac{20}{100}$	5,100
Total annual cost (excluding item cost)	10,200

Increase in annual cost by adopting (A) above : Rs. (71,650 – 10,200) = Rs. 61,450.

Amount of quantity discount: 5% × Rs. 500 × 5,200, units = Rs. 1,30,000.

Since the amount of quantity discount (Rs. 1,30,000) is more than the increase in total annual cost (Rs. 61,450), it is advisable to accept the offer. This will result in a saving of Rs. (1,30,000 - 61,450) or Rs. 68,550 p.a. in inventory cost.

(2) Maximum Level of Stock:

= Re-order level + Re-order quantity – (Minimum usage × Minimum delivery period) = 1,600 units + 102 units – (50 units × 6 weeks) = 1,402 units.

[Assume that the Reorder quantity is supplied as soon as the Reorder level is reached]

(3) Minimum Level of Stock:

= Re-order level – (Normal usage × Normal delivery period) [see Note] = 1,600 units – (100 units × 7 weeks)

= 900 units. Note: Normal delivery period is taken to be the average delivery period.

(4) Re-order Level of Stock:

= Maximum usage × Maximum delivery period = 200 units × 8 weeks = 1,600 units.

Answer for Q.NO.18.

(a) Optimum run size or Economic Batch Quantity (EBQ)

$$=\sqrt{\frac{2\times AnnualOutput\times Setupcost}{Annualcost\,of\,Carrying\,one\,unit}}=\sqrt{\frac{2\times 24000\times 324}{0.10\times 12}}=3600units$$

(b) Interval between two consecutive optimum runs = $\frac{FBQ}{MonthlyOutput} \times 30$

$$= \frac{3600}{24000 \div 12} \times 30 = 54 \text{ Calendar days}$$

(c) Minimum inventory holding cost = Average inventory × Annual carry-ing cost of one unit of inventory

= (3600 ÷ 2) × 0.10×12 = Rs. 2,160.

<u>Answer for Q.NO.19.</u>

The total cost of the three locations are:

At Total cost = Fixed cost + Variable cost for a volume "X"

Patna => Total cost = 30,00,000 + 300 × X

Ranchi => Total cost = 50,00,000 + 200 × X

Dhanbad => Total cost = 25,00,000 + 350 × X

We can compute and plot the total costs per annum at the three different locations for the various cases of production volume of 5,000, 10,000, 15,000, 20,000 25,000 units.

(i) Patna

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	30,00,000	30,00,000	30,00,000	30,00,000	30,00,000
Variable Cost (Rs. 300 x)	300 (5,000)	300 (10,000)	300 (15,000)	300 (20,000)	300 (25,000)
Total Cost (Rs.)*	= Rs.45 lakhs	= Rs.60 lakhs	= Rs.75 lakhs	= Rs.90 lakhs	= Rs.105 lakhs

(ii) Ranchi

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	50,00,000	50,00,000	50,00,000	50,00,000	50,00,000
Variable Cost (Rs. 200 x)	200 (5,000)	200 (10,000)	200 (15,000)	200 (20,000)	200 (25,000)
Total Cost (Rs.)*	= Rs.60 lakhs	= Rs.70 lakhs	= Rs.80 lakhs	= Rs.90 lakhs	= Rs.100 lakhs

(iii) Dhanbad

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	25,00,000	25,00,000	25,00,000	25,00,000	25,00,000
Variable Cost (Rs. 300x)	350 (5,000)	350 (10,000)	350 (15,000)	350 (20,000)	350 (25,000)
Total Cost (Rs.)*	= Rs.42.5 lakhs	= Rs.60 lakhs	= Rs.77.5 lakhs	= Rs.95 lakhs	= Rs.112.5 lakhs

* In all the above tables, Total Cost = Fixed Cost + Variable Cost

If the volume distribution be as follows:

	Up to 10,000 units	Between 10,000 units to 20,000 units	Above 20,000 units
Favourable Location	Dhanbad	Patna	Ranchi

For a volume of 18000 units favourable location is Patna which can be substantiated by the followings calculations

of Total Cost :-

Patna => 30,00,000 + 300 × 18,000 = Rs.84 lakhs

Ranchi => 50,00,000 + 200 × 18,000 = Rs.86 lakhs

Dhanbad => 25,00,000 + 350 × 18,000 = Rs.88 lakhs.

Answer for Q.NO.20.

Calculation of EBQ:

$$\text{EBQ} = \sqrt{\frac{2 \times \text{Annual Demand} \times \text{Set-up cost}}{\text{Unit Cost} \times \text{Inventory carrying cost per unit per year } (\bullet)}}$$

$$= \sqrt{\frac{(2 \times 12 \times 1000 \times 120)}{(0.1 \times 20)}} = 1200 \text{units}$$

Answer for Q.NO.21.

We can call the years as ‘X’ and exports as ‘Y. In order to use the normal equations for the least square line, we need ΣX, ΣY, ΣXY and ΣX2. If we arrange X in such a way that ΣX = 0, it will simplify our calculations. Therefore, we call the year 2008 as 0, 2007 as -1 and 2009 as + 1 and likewise for the other years in the data.

The rearrangement is shown in the table as follows:

X	Y	X2	XY
-4	13	16	-52
-3	20	9	-60
-2	20	4	-40
-1	28	1	-28
0	30	0	0
1	32	1	32
2	33	4	66
3	38	9	114
4	43	16	172

ΣX = 0 ΣY = 257 Σx2 = 60 ΣXY - 204

Let the equation of the best fit straight line to the given data be = + 0 1 Y a ax

So the normal equations are

ΣY = aoN + a1ΣX .... (1)

$\Sigma XY = ao\Sigma X + a1\Sigma X2$ .... (2)

As $\Sigma X = 0$, from (1) $\Sigma Y = aoN$ from (2) $\Sigma XY = a_1 \Sigma X^2$

Therefore, $a_o = \Sigma Y / N = 257 / 9 = 28.56$ [N = No. of years]

$a_1 = \Sigma XY / \Sigma X^2 = 204 / 60 = 3.4$

The equation of a straight line fitting the data is:

$Y = 28.56 + 3.4 X$

(a) Forecast for 2013, (i.e., X = 5): Y = 28.56 + 3.4 (5) = 45.56 ('000) nos.

(b) Forecast for 2014, (i.e., X = 6): Y = 28.56 + 3.4 (6) = 48.96 ('000) nos.

Answer for Q.NO.22.

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 1000 \times 5}{1.25}} = \sqrt{8000} = 89.44 \text{units}$$

Re-order unit = dL = $\frac{1000}{365} \times 5 = 13.7 \text{units}$

Total Cost = $DC + \frac{D}{Q} \times S + \frac{Q}{2} \times H = 1000 \times 12.5 + (1000/89.44) \times 5 + (89.44/2) \times 1.25$

= Rs. 2611.81

Answer for Q.NO.23.

d = D/no. week days = 1000/250 = 4

Re-order level(R) = dL + z L = 4×15 + 1.64 × 25 = 101

Answer for Q.NO.24.

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times (60 \times 365)}{0.5}} = \sqrt{876000} = 936 \text{units}$$

$$\sigma_1 = \sqrt{\sum_{i=1}^{L} \sigma_d^2} = \sqrt{6 \times 7^2} = 17.15$$

Re-order leve(R) = dL + z 6L = 60×6 + 1.64 × 17.15 = 188

Answer for Q.NO.25.

$Q = d(T+L) + z\sigma_{T+L} - I = 10\ (30+14) + \tau\ \sigma_{T+L} - 150 =$

$$\sigma_{T+L} = \sqrt{\sum_{i+1}^{T+L} \sigma_d^2} = \sqrt{(T+L)\sigma_d^2} = \sqrt{(30+40) \times 3^2} = 19.90$$

T for P = 0.98 is 2.05

Q = 160 – 150 + 2.05×19.9 = 331 units

Answer for Q.NO.26.

Avg. Inventory = Q/2 + SS = 150 + 40 =190

Inventory Turn = $\frac{D}{\frac{Q}{2} + SS} = \frac{1000}{190} = 5.263 \text{ternper year}$

Answer for Q.NO.27.

Avg. Inventory = dT/2 + SS = (50 × 3)/2 + 30 = 105 units

Inventory Turn = $\frac{52d}{\text{Avg.Inventory}} = \frac{52 \times 50}{105} = 24.8 \text{ turnsper year}$

= 24.8 turns per year

Answer for Q.NO.28.

$$EOQ1 = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 5}} = 63.24$$

$$EOQ2 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 4.5}} = 66.67$$

$$EOQ3 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 3.9}} = 71.6$$

Total Cost1 = $DC + \frac{D}{Q} \times S + \frac{Q}{2} \times iC = 56323$

TC2 = 51000

TC3 = 44585.69

1000 units should be ordered.

Answer for Q.NO.29.

The cost of underestimating the demand is loss of profit (Cu) or 100-70=30/unit. The cost of overestimating demand is the loss in occured when the unit must be sold at salvage value(Co)

= 70 – 20 = 50

The optimal prob. Of not being sold

P<= Cu/Co + Cu = 30/30 + 50 = 0.375

From the data, this corresponds to 37th value.

No. of unit sold

Unit demand	Prob.	35	36	37	38	39	40
35	0.1	0	50	100	150	200	250
36	0.15	30	0	50	100	150	200
37	0.25	60	30	0	50	100	150
38	0.25	90	60	30	0	50	100
39	0.15	120	90	60	30	0	50
40	0.1	150	120	90	60	30	0
Total	1	75	53	43	53	83	125

Answer for Q.NO.30.

$S.D = \sqrt{21(5) \times (5)} = 23$

Z = 1.64

$q = d\times(T + L) + Z\times S.D - I$

$= 20(14 + 7) + 1.64\times23 - 180$

$= 278$ units

Answer for Q.NO.31.

Classification	Item no.	Annual Rupee Usage	% of total
A	22,68	1,70,000	72.9%
B	27,03,82	53,000	22.7%
C	54,36,19,23,41	10,450	4.5%

4. APPLICATION OF OPERATION RESEARCH - PRODUCTION PLANNING AND CONTROL

Answer for Q.NO.1.

Let x_1 be the no. of units of A

Let x_2 be the no. of units of B

Objective function: Min.Z = 4x1 + 6x2

Subject to Constraints:

$x_1+2x_2 \geq 8$ (Constraint on requirement of ingradient C)

$3x_1+x_2 \geq 75$ (Constraint on requirement of ingradient D)

And $x_1, x_2 \geq 0$ (No negativity constraint)

Answer for Q.NO.2.

Shares	Dividend	Growth in Rs.
A	12%	10/100 = 0.1
B	4%	40/100 = 0.4
Min-income	600	1000

Let x_1 be the amount invested on share A

Let x_2 be the amount invested on share B

Objective function: Min. Z = x1+ x2

Subject to constraints:

$0.12\, x_1 + 0.04\, x_2 \geq 600$ (Dividend income constraint)

$0.1\, x_1 + 0.4\, x_2 \geq 1000$ (Growth constraint)

And $x_1, x_2 \geq 0$. (Non negativity constraint)

Answer for Q.NO.3.

Let α be no. of hours of plant A in use

Let β be no. of hours of plant B in use

Objective function: Min $Z = 9\alpha + 10\beta$

Subject to constraints:

$2\alpha + 4\beta \geq 50$ (Constraint relating to Product X)

$4\alpha + 3\beta \geq 24$ (Constraint relating to Product Y)

$3\alpha + 2\beta \geq 60$ (Constraint relating to Product Z)

And $\alpha, \beta \geq 0$ (Non negativity constraint)

Answer for Q.NO.4.

Let x1 be the no. of units of product P

Let x2 be the no. of units of product Q

Let x3 be the no. of units of product R

Objective function: Max. $Z = 3x_1+5x_2 + 4x_3$

Subject to constraints:

$2x_1+3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')

$3x_1+2x_2 + 4x3 \leq 15$ (Constraint on availability of Raw Material 'B')

$2x_2+5x_3 \leq 10$ (Constraint on availability of Raw Material 'C')

And x1, x2, x3 ≥ 0 (Non negativity constraint)

Answer for Q.NO.5.

Let x_1 be the amount allocated for personal loan

Let x_2 be the amount allocated for car loan

Let x_3 be the amount allocated for Housing loan

Let x_4 be the amount allocated for agricultural loan

Let x_5 be the amount allocated for Commercial loan

Objective Function: Max Z

$= 0.17x_1+ 0.14x_2+ 0.11x_3+ 0.1x_4+ 0.13x_5 – (0.10x_1+ 0.07x_2+ 0.05x_3+ 0.08x_4+ 0.06x_5)$

$= (0.17 - 0.10)x_1+ (0.14 - 0.07)x_2 + (0.11 - 0.05)x_3 + (0.10 - 0.08)x_4+ (0.13 – 0.06)x_5$

$= 0.17x_1+ 0.07x_2 + 0.06x_3 + 0.02x_4+ 0.07 x_5$

Subject to constraints

(i) $x_1+ x_2+ x_3+ x_4 + x_5 \leq 600$ Millions (Constraint on total loan amount)

(ii) $x_4 + x_5 \geq 0.4 (x_1+ x_2+ x_3+ x_4 + x_5)$ (Constraint due to policy set for Agricultural and Commercial Loan)

(iii) $x_3 \geq 0.5 (x_1+ x_2+ x_3)$ (Constraint due to policy set for Housing Loan)

(iv) $0.1x_1 + 0.07x_2+ 0.05x_3+ 0.08x_4+ 0.06 x_5 \leq 0.06$ Million (Constraint on limit of overall bad debt)

(v) $x_1, x_2, x_3, x_4, x_5 \geq 0$ (Non negativity constraint)

Answer for Q.NO.6.

Let x_1 be the chairs produced of A type

x_2 be the chairs produced of B type

x_3 be the chairs produced of C type

Objective function

Maximise $Z = 40x_1 + 35x_2 + 30x_3$

Subject to constraints:

$2x_1 + 3x_2 + 2x_3 \leq 120$ (Constraint on available time of 1st year class)

$4x_1 + 3x_2 + x_3 \leq 160$ (Constraint on available time of 2nd year class)

$3x_1 + 2x_2 + 4x_3 \leq 100$ (Constraint on available time of 3rd year class)

$x_1, x_2, x_3 \geq 0$ (Non negativity constraint)

Answer for Q.NO.7.

DATA SUMMARY CHART

Decision variables	**Products**	**Type of raw material**			**Profits per unit**
		A	**B**	**C**	**(Rs.)**
x_1 x_2 x_3	P Q R	2 - 3	3 2 2	- 5 4	3 5 4
Units of material available:		8 Maximum	10 maximum	15 maximum	

x_1 = number of units of Product P

x_2 = number of units of Product Q

x_3 = number of units of Product R

The given Q is formulated as the LP model as follows:

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints :

$2x_1 + 3x_3 \leq 8$ (Constraint due to availability of Material A)

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint due to availability of Material B)

$5x_2 + 4x_3 \leq 15$ (Constraint due to availability of Material C)

$x_1, x_2, x_3, \geq 0$ (Non negativity constraint)

Answer for Q.NO.8.

$x, + x_3 \geq 15$, $x_3 + x_4 \geq 8$, $x_4 + x_5 \geq 20$, $x_5 + x_6 \geq 6$, and $x_6 + x_1 \geq 2$.

Since, the objective is to minimize the total number of nurses employed in the hospital,

$Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

Obviously, we must have $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$.

Answer for Q.NO.9.

Step 1. The appropriate mathematical formulation of the given Q. . is as follows:

Maximize (total effective audience) $Z = 40{,}000\ x_1 + 55{,}000\ x_2$

Subject to the constraints

$1{,}000x_1 + 1{,}500x_2 \leq 20{,}000$ (Budget constraint)

$x_1 \leq 12$ (Constraint on annual no. of insertions in Media A)

$x_1 \geq 5$ or $-x2 \leq -5$ (Constraint on annual no. of insertions in Media B)

$x_1, x_2 \geq 0$ (Non negativity constraint)

where

x_1 = annual number of insertions/messages for media A.

x_2 = annual number of insertions/ messages for media B.

Answer for Q.NO.10.

i. The mathematical formulation of the linear programming problem is

Maximise $Z = 7x_1 + 5x_2$

Subject to $3x_1 + x_2 \leq 48$

$2x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$

Where x_1 and x_2 denote the number of units of product A and B respectively.

ii. The dual of the above problem is:

Minimize $Z^* = 48y_1 + 40y_2$

$3y_1 + 2y_2 \geq 7$

$y_1 + y_2 \geq 5$

$y_1, y_2 \geq 0$

Where y1 and y2 are the dual variables indicating the shadow prices of raw material and labour respectively

Answer for Q.NO.11.

Raw Materials	x1	x2	x3	Available units
	P	Q	R	
A	2	-	3	8
B	3	2	2	10
C	-	5	4	15
	3	5	4	

Profits 3/- 5/- 4/-

Let x_1 be the no. of units of P

Let x_2 be the no. of units of Q

Let x_3 be the no. of units of R

Objective function: Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to constraints:

$2x_1 + 3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint on availability of Raw Material 'B')

$5x_2 + 4x_3 \leq 15$ (Constraint on availability of Raw Material 'C')

And $x_1, x_2, x_3 \geq 0$. (Non negativity constraint)

Primal

Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$2x_1 + 3x2 \leq 8$

$3x_1 + 2x_2 + 2x_3 \leq 10$

$5x_2 + 4x_3 \le 15$

And $x_1, x_2, x_3 \ge 0$

Dual

Min. $Z = 8y_1 + 10y_2 + 15y_3$

Subject to

$2y_1 + 3y_2 \ge 3$

$3y_1 + 2y_2 + 5y_3 \ge 5$

$2y_2 + 4y_3 \ge 4$

And $y_1, y_2, y_3 \ge 0$

$2x_1 + 3x_2 + S_1 = 8$

$3x_1 + 2x_2 + 2x_3 + S_2 = 10$

$5x_2 + 4x_3 + S_3 = 15$

Max $Z = 3x_1 + 5x_2 + 4x_3 + 0.S_1 + 0.S_2 + 0.S_3$

$\therefore x_1 = 23/20 \; x_2 = 19/10 \; x_3 = 11/8$

$Z = 18.45$

Answer for Q.NO.12.

Let x_1 be the no. of units of product A

Let x_2 be the no. of units of product B

Let x_3 be the no. of units of product C

Let x_4 be the no. of units of product D

Objective function Maximize $Z = 5x_1+7x_2+3x_3+9x_4$

	A	B	C	D	Supply in Kgs.
I type material	4	3	8	2	800
II type material	1	2	0	1	300
Machine	8	5	0	4	500
Labour	3	2	1	5	900
Profit	5	7	3	9	

Subject to constraints

$4x_1+3x_2+8x_3+2x_4 \le 800$ (Constraint on availability of Material type I)

$x_1+2x_2+0.x_3+x_4 \le 300$ (Constraint on availability of Material type II)

$8x_1+5x_2+0.x_3+4x_4 \le 500$ (Constraint on Machine Hours available)

$3x_1+2x_2+x_3+5x_4 \le 900$ (Constraint on Labour Hours available)

and $x_1, x_2, x_3, x_4 \ge 0$. (Non negativity constraint)

Answer for Q.NO.13.

Let x_1, x_2, x_3, x_4, x_5 and x_6 represent the six different investment alternatives, i.e., x_1 is bank deposit, x_2 is treasury note. X_3 corporate deposit, X_4 blue chip stock, X_5 speculative stock and x_6 real estate. The objective is to maximize the annual yield of the investors (in number of units) given by the linear expression.

Maximize Z = $9.5x_1 + 8.5x_2 + 12.0x_3 + 15.0x_4 + 32.5x_5 + 35.0x_6$

Subject to the Constraints:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 1$ (Investment decision)

$0.02x_1 + 0.01x_2 + 0.08x_3 + 0.25x_4 + 0.45x_5 + 0.40x_6 \leq 0.20$ (Constraint on weighted average risk of the portfolio)

$6x_1 + 4x_2 + 3x_3 + 5x_4 + 3x_5 + 10x_6 \geq 5$ (Constraint on weighted average length of period of investment)

$x_5 + x_6 \leq 0.25$ (Constraint on investment in real estate and speculated stock)

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (non-negativity condition)

<u>Answer for Q.NO.14.</u>

The slope form is $Y = mX + b$ where m = slope

Rearranging,

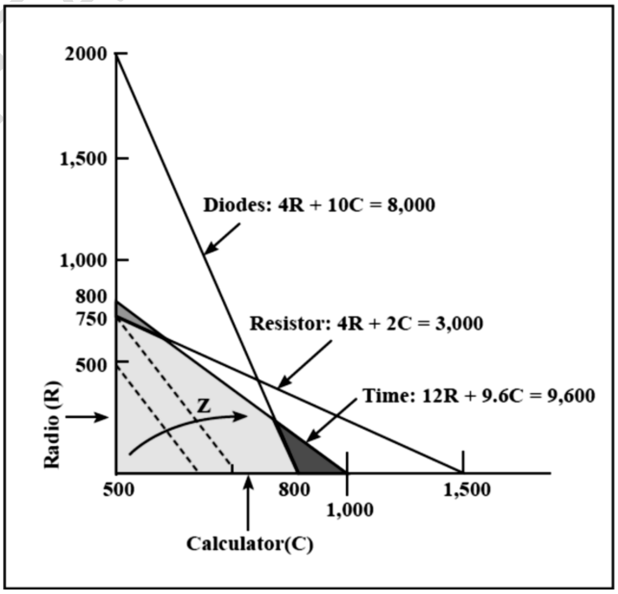
45Y = – 15X + Z

$$Y = -\frac{15x}{45} + \frac{z}{45}$$

Slope is –15/45 or – 1/3.

<u>Answer for Q.NO.15.</u>

The decision variables are radios, R, and calculators, C, and we must determine how many of each should be produced to maximize profit, Z.



(1) Objective function

Max Z= 10R + 15C

Constraints

Diodes (8,000 available): Radios require 4 each, and calculators require 10 each.

$$\therefore 4R + 10C \leq 8{,}000$$

Resistors (3,000 available): Radios require 4 each, and calculators require 2 each.

$$\therefore 4R + 2C \leq 3{,}000$$

Testing (9,600 minutes available): Radios require 12.0 minutes, and calculators require 9.6 minutes.

$$12R + 9.6C \leq 9{,}600$$

(2) Graph ‘bf variables and constraints

Plotting each of the constraints inequality as an equality, we have:

For Diodes: 4R + 10C = 8000

If R = 0, then C = 800

If C = 0, then R = 2,000

For Resistors: 4R + 2C = 3,000

If R = 0, then C = 1500

If C = 0, then R = 750

For Testing: 12R + 9.6C = 9,600

If R = 0, then C = 1,000

If C = 0, then R = 800

Note: The resulting graph establishes a feasible region bounded by the time, diode, and resistor constraints that R ≥ 0 and C ≥ 0.

(3) Slope of the objective function.

We can express our objective function in slope intercept from, where the Y axis corresponds to R and the X axis to C.

Z = 10R + 15C

Or, 10R = -15C + Z

$$\therefore R = -\frac{15}{10}C + \frac{z}{10} = -\frac{3}{2}C + \frac{z}{10}$$

∴ Slope = – 3/2, which means that for every 3-unit decrease in Y there is a 2 increase in *X*. This slope is plotted as a dotted line in the graph by marking off 3 units (negative) in *R* for each 2 units (positive) in *C*.

(4) Move objective function to optimize. The slope of the objective function hi to objective line) is moved away from the origin until constrained. In this case the binding constraints are the diode inventory supply and testing machine time availability.

(5) Read solution values. The arrows point to the approximate R and C coordinates o the constraining intersection.

Number of radios = 240

Number of calculators = 700

Note: That the simultaneous solution of the two binding constraint equations would lend more accuracy to the answer:

(4R + 10C = 8,000) × (-3) = – 12R – 30C = – 24,000

$$\underline{12R + 9.6C = 9{,}600}$$

$$-20.4C = -14{,}400$$

$$C = 705 \text{ calculators}$$

Substituting to solve for R:

4R + 10(705) = 8,000

∴ R = 8,000 – 7,050/ 4 = 237 radios

Comment: We had two decision variables (that is, products) to choose from and established a profit function, Z, and constraints and optimized the function by moving it away from the origin. The graph of this example showed that the resistor supply was not constraining, so only two constraints (diodes and test time) were binding. Similarly, there were two decision variables in the solution, that is, we ended up producing both radios and calculators. The number of variables in solution will always equal the number of explicit constraints that are binding.

The graphic linear programming solution gives an indication of the sensitivity of the solution to changes in the constraints. If for example, additional diodes could be purchased from an outside supplier with no increase in cost, profit would be maximized by extending the iso – objective line to the next corner and producing 1,000 calculators and no radios. In this case we would have one explicit constraint (time) binding and only one decision variables (calculators) in the final solution.

Answer for Q.NO.16.

(a) Objective function Max Z = 5X + 20Y

Constraints:

Cleaning 2X + 4Y ≤ 10

Testing 6X + 3Y ≤ 12

(b)

C →		5	20	0	0	
↓ Variables in solution		Decision variables				Solution Values (RHS)
		X	Y	S_1	S_2	
0	S_1	2	4	1	0	10
0	S_2	6	3	0	1	12
	Z	0	0	0	0	0
	C-Z	5	20	0	0	

Answer for Q.NO.17.

(a) 5A + 9B ≤ 36, 8B + 5C ≤ 24, and 2A + 5C ≤ 7

(b) Three

(c) Max Z = 4A + 8B + 6C

Answer for Q.NO.18.

(i) Table: 1 Cost Matrix

From \ To	C_1	C_2	C_3	Supply
M_1	8	7	6	250
M_2	5	4	9	320
M_3	7	5	5	280
Demand	300	260	180	850 / 750

From the given data we have Total Supply = 850 tonnes and total and total Demand = 740 tonnes i.e., Supply ≠ Demand.

So this is an unbalanced problem of transpartation. To make it balanced we introduce a "Dummy" construction site of demand 850 – 740 = 110 tonnes and having zero cost elements for all the cells of the matrix corresponding to it.

Table: 2 Basic Feasible Solution by VAM (Optimal)

From \ To		C_1	C_2	C_3	Dummy	Supply	Row Penalties 1st	2nd	3rd	4th	Row Nos. (u_i)
M_1		8	7	(140) 6	(110) 0	250 ~~140~~	6*	2	1	1	$u_1 = 0$
M_2		(300) 5	(20) 4	9	0	320 ~~20~~	4	1	5*	–	$u_2 = -2$
M_3		7	(240) 5	(40) 5	0	280 ~~40~~	5	0	0	0	$u_3 = -1$
Demand		~~300~~	~~240~~ ~~260~~	~~40~~ ~~180~~	~~110~~	850					
Column Penalties	1st	2	1	1	0						
	2nd	2*	1	1	–						
	3rd	–	1	1	–						
	4th	–	2*	1	–						
Column Nos. (v_j)		$v_1 = 7$	$v_2 = 6$	$v_3 = 6$	$v_4 = 0$						

Row Penalty = 2nd lowest cost figure of a row – Lowest cost figure of that row.

For the **1st Set of Row Penalties –**

(a) For 1st row, 2nd lowest cost = 6 and lowest cost = 0

∴ Penalty = 6 – 0 = 6

(b) For 2nd Row, 2nd lowest cost = 4 and Lowest cost = 0

∴ Penalty = 4 – 0 = 4

(c) For 3rd Row, 2nd lowest cost = 5 and Lowest cost = 0,

∴ Penalty = 5 – 0 = 5

Similarly, **Column Penalty** = 2nd lowest cost figure of a column – Lowest cost figure of that column

For the **1st Set** of **Column Penalties** –

(a) For 1st column, 2nd lowest cost = 7 and Lowest cost = 5, ∴ Penalty = 7 – 5 = 2

(b) For 2nd column, 2nd lowest cost = 5 and Lowest cost = 4, ∴ Penalty = 5 – 4 = 1

(c) For 3rd column, 2nd lowest cost = 6 and Lowest cost = 5, ∴ Penalty = 6 – 5 = 1

Of all these Row and Column penalties of 1st set, 6 is highest and it corresponds to 1st Row.

Hence allocation should be done at that cell of 1st Row where cost is least. This corresponds to the cell (M1 –

Dummy). So maximum possible unit of 110 is allocated in this cell by maintaining parity of supply and demand.

With this allocation the total demand of 'Dummy' site is exhausted. But the supply of the corresponding Mineral Field (M1) is not fully exhausted. Remaining supply capacity of M1 i.e. 250 –

110 = 140 tonnes is shown as balance in the supply cell of M1. As the demand of 'Dummy' is fullfilled, the entire column for this has been shaded indicating the same. Figures of this column will no longer participate in any of the subsequent calculations of Penalty (for Rows as well as columns)

The same procedure of calculating penalty for Rows and Columns and subsequently allocating maximum possible quantity in the least cost cell corresponding to highest penalty is repeated until all the allocations are made maintaining parity of Supply and Demand.

The solution thus obtained is the **Basic Feasible Solution**. It is given as follows.

Table: Showing Optimum Allocation

Cell	Allocation	Cost of Transportation (Rs.)
M1 - C3	140 tonnes	140 × 6 = 840
M1 - Dummy	110 tonnes	110 × 0 = 0
M2 - C1	300 tonnes	300 × 5 = 1500
M2 - C2	20 tonnes	20 × 4 = 80
M3 - C2	240 tonnes	240 × 5 = 1200
M3 - C3	40 tonnes	40 × 5 = 200
Total	850 tonnes	Rs. 3820

Here, m = No. of rows of the matrix = 3

n = No. of columns of the matrix = 4

∴ m + n – 1 = 3 + 4 – 1 = 6

Also, no. of allocated cells = 6

As, no. of allocated cells = 6 = m + n – 1, the solution is a **non degenerate** one.

Now the solution is tested for **OPTIMALITY**.

For this, Row Nos. (u_i) and column nos. (v_j) are calculated by using the equation $C_{ij} = u_i + v_j$ for all the **allocated cells**, where C_{ij} = Cost figure of the cell i-j.

Allocated Cell	C_{ij}	$C_{ij} = u_i + v_j$			
$M_1 - C_3$	$C_{13} = 6$	$C_{13} = u_1 + v_3$	or, $6 = u_1 + v_3$	or, $6 = 0 + v_3$ (Assume $u_1 = 0$) or, $v_3 = 6$	**(1)**
M_1 - Dummy	$C_{14} = 0$	$C_{14} = u_i + v_4$	or, $0 = u_1 + v_4$	or, $0 = 0 + v_4$ or, $v_4 = 0$	**(2)**
$M_2 - C_1$	$C_{21} = 5$	$C_{21} = u_2 + v_1$	or, $5 = u_2 + v_1$	or, $5 = -2 + v_1$ or, $v_1 = 7$	**(6)**
$M_2 - C_2$	$C_{22} = 4$	$C_{22} = u_2 + v_2$	or, $4 = u_2 + v_2$	or, $4 = u_2 + 6$ or, $u_2 = -2$	**(5)**
$M_3 - C_2$	$C_{32} = 5$	$C_{32} = u_3 + v_2$	or, $5 = u_3 + v_2$	or, $5 = -1 + v_2$ or, $v_2 = 6$	**(4)**
$M_3 - C_3$	$C_{33} = 5$	$C_{33} = u_3 + v_3$	or, $5 = u_3 + v_3$	or, $5 = u_3 + 6$ or, $u_3 = -1$	**(3)**

Hence no. of equations = 6 and no. of unknowns = 7. So to start with a solution, it is assumed u1 = 0. Thereafter all the other row nos. and column nos. are calculated. The sequence of usage of the above equations is indicated as (1), (2), (3), ... (6).

Next opportunity cost (Δ_{ij}) for all the unallocated cells are calculated using $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Unallocated Cell	C_{ij}	Opportunity Cost [$\Delta_{ij} = C_{ij} - (u_i + v_j)$]
M_1 - C_1	$C_{11} = 8$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 8 - (0 + 7) = 1$
M_1 - C_2	$C_{12} = 7$	$\Delta_{12} = C_{12} - (u_1 + v_2) = 7 - (0 + 6) = 1$
M_2 - C_3	$C_{23} = 9$	$\Delta_{23} = C_{23} - (u_2 + v_3) = 9 - (-2 + 6) = 5$
M_2 - Dummy	$C_{24} = 0$	$\Delta_{24} = C_{24} - (u_2 + v_4) = 0 - (-2 + 0) = 2$
M_3 - C_1	$C_{31} = 7$	$\Delta_{31} = C_{31} - (u_3 + v_4) = 7 - (-1 + 7) = 1$
M_3 - Dummy	$C_{34} = 0$	$\Delta_{34} = C_{34} - (u_3 + v_4) = 0 - (-1 + 0) = 1$

As all the opportunity cost values are nonnegative, the solution is optimal.

(i) So the optimal transportation plan is as shown in Table-3 and minimum cost of transportation is Rs. 3820/-

(ii) Quantities to be produced by M1, M2 and M3 are respectively 250,320 and 280 tonne of which 110 tonnes worth of stone chips produced by M1 will remain unused by the construction sites. So this quantity can be sold ex-field.

Answer for Q.NO.19.

Table: 1 Profit Matrix

Manufacturer \ Dress Size	I	II	III	IV	Supply
A	2.5	4	5	2	150
B	3	3.5	5.5	1.5	450
C	2	4.5	4.5	2.5	250
Demand	100	200	450	150	850 / 900

Maximum possible supply capability of manufacturer = 850 units

Total Demand = 900 units

As Supply ≠ demand, the problem is an unbalanced one. To make it balanced, a 'Dummy' manufacturer of supply capacity = 900 – 850 = 50 units. is introduced. The profit figures for it are all zeros.

Also it is a problem of maximisation, to convert it to a problem of minimisation, a Relative Loss matrix is formed by subtracting all the profit figures given in the above matrix as well as those of Dummy from the highest profit (5.5) figure of the given matrix.

Table : 2 Relative Loss Matrix with Basic Feasible Solution

Manufacturer \ Dress Size	I	II	III	IV	Supply	Row Penalties 1st	2nd	3rd
A	100 3	1.5	0.5	50 3.5	150 ~~50~~	1	1.5	0.5*
B	2.5	2	450 0	4	~~450~~	2*	–	–
C	3.5	200 1	1	50 3	~~250~~ ~~50~~	0	2*	0.5
Dummy	5.5	5.5	5.5	50 5.5	~~50~~	0	0	0
Demand	~~100~~	~~200~~	~~450~~	~~150~~	900			
Column Penalties 1st	0.5	0.5	0.5	0.5				
2nd	0.5	0.5	–	0.5				
3rd	0.5	–	–	0.5				

Here, m = No. of rows of the matrix = 4 and n = No. of columns of the matrix = 4

∴ m + n – 1 = 4 + 4 – 1 = 7

Also no. of allocated cells = 6 ≠ (m+ n – 1)

So the solution is a degenerate one. To resolve this, we make use of an artificial quantity 'e' and allocate this quantity at the unallocated cell which is having least cost among all the unallocated cells. It can be mentioned that the quantity 'e' is very small and for all practical purposes its value can be taken as zero.

Least cost unallocated cell is (A-III) where allocation of 'e' has to be made.

Table : 3 Showing Basic Feasible Solution (Optimal)

Manufacturer \ Dress Size	I	II	III	IV	Supply	Row Nos. (u_i)
A	100 3	1.5	3 0.5	50 3.5	150	$u_1 = 0$
B	2.5	2	450 0	4	450	$u_2 = -0.5$
C	3.5	200 1	1	50 3	250	$u_3 = -0.5$
DUMMY	5.5	5.5	5.5	50 5.5	50	$u_4 = 2$
DEMAND	100	200	450	150	900	
Column Nos. (v_j)	$v_1 = 3$	$v_2 = 1.5$	$v_3 = 0.5$	$v_4 = 3.5$		

To test optimality of the Basic Feasible Solution, Row Nos. (ui) and Column Nos. (vj) are calculated using the equation Cij = ui + vj for the allocated cells, where Cij = Relative Loss figure of the cell i - j.

Allocated cell	A-I	A-III	A-IV	B-III	C-II	C-IV	Dummy-IV
C_{ij}	$C_{11}= 3$	$C_{13}= 0.5$	$C_{14}= 3.5$	$C_{23}= 0$	$C_{32}= 1$	$C_{34}= 3$	$C_{44}= 5.5$

$C_{11} = u_1 + v_1$ or, $3 = 0 + v_1$ [$u_1 = 0$, Assumed] or, $v1 = 3$

$C_{13} = u_1 + v_3$ or, $0.5 = 0 + v_3$ or, $v_3 = 0.5$; $C_{14} = u_1 + v_4$ or, $3.5 = 0 + v_4$ or, $v_4 = 3.5$

$C_{23} = u_2 + v_3$ or, $0 = u_2 + 0.5$ or, $u_2 = -0.5$; $C_{34} = u_3 + v_4$ or, $3 = u_3 + 3.5$ or, $u_3 = -0.5$

$C_{32} = u_3 + v_2$ or, $1 = -0.5 + v_2$ or, $v_2 = 1.5$; $C_{44} = u_4 + v_4$ or, $5.5 = u_4 + 3.5$ or, $u_4 = 2$

Opportunity Loss figures (Δif) for all the unallocated cells are calculated using the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Unallocated Cell	Opportunity Loss (Δij)
A - II	$\Delta_{12} = C_{12} - (u_1 + v_2) = 1.5 - (0 + 1.5) = 0$
B - I	$\Delta_{21} = C_{21} - (u_2 + v_1) = 2.5 - (-0.5 + 3) = 0$
B – II	$\Delta_{22} = C_{22} - (u_2 + v_2) = 2 - (-0.5 + 1.5) = 1$
B - IV	$\Delta_{24} = C_{24} - (u_2 + v_4) = 4 - (-0.3 + 3.5) = 1$
C - I	$\Delta_{31} = C_{31} - (u_3 + v_1) = 3.5 - (-0.5 + 3) = 1$
C - III	$\Delta_{33} = C_{33} - (u_3 + v_3) = 1 - (-0.5 + 0.5) = 1$
Dummy - I	$\Delta_{41} = C_{41} - (u_4 + v_1) = 5.5 - (2 + 3) = 0.5$
Dummy - II	$\Delta_{42} = C_{42} - (u_4 + v_2) = 5.5 - (2 + 1.5) = 2$
Dummy - III	$\Delta_{43} = C_{43} - (u_4 + v_3) = 5.5 - (2 + 1.5) = 3$

As all the opportunity loss values are non negative, the solution is optimal.

Table Showing Optimum allocation of orders quantities

From Manufacturer	Dress Size	Allocated Quantity	Profit/unit (Rs.)	Total (Rs.)
(i)	(ii)	(iii)	(iv)	(v) = (iii) × (iv)
A	I	100 units	2.5	250
	IV	50 units	2	100
B	III	450 units	5.5	2475
C	II	200 units	4.5	900
	IV	50 units	2.5	125
Dummy	IV	50 units	0	0
Total	–	**900 units**	–	**Rs. 3850**

Maximum Profit = Rs. 3850/-

Answer for Q.NO.20.

(a) From the given data total plant capacity (1500 units) is more than the total demand of warehouses (1400 units). So the problem is unbalanced. To make it balanced, a 'Dummy' warehouse of demand 1500 – 1400 = 100 units is introduced. Cost figures corresponding to various cells of this 'Dummy' are zeros.

Table : 1 Basic Feasible Solution

Plant \ Warehouse		W$_1$	W$_2$	W$_3$	W$_4$	W$_5$	Dummy	Plant Capacity	Raw Penalities 1	2	3	4	5	6
F$_1$		74	56 (280)	54	62 (120)	68	0	~~120~~ ~~400~~	54	2	6	6	6	6
F$_2$		58 (200)	64	62	58	54 (200)	0 (100)	~~400~~ ~~500~~ ~~200~~	* 54	4	4	4	4	-
F$_3$		66	70	52 (240)	60 (240)	60 (120)	0	~~600~~ ~~360~~ ~~240~~	52	* 8	0	0	0	0
Warehouse Demand		~~200~~	~~280~~	~~240~~	~~360~~	~~120~~ ~~320~~	~~100~~	1500						
Column Penalities	1	8	8	2	2	6	0							
	2	8	8	2	2	6	-							
	3	8	8*	-	2	6	-							
	4	8*	-	-	2	6	-							
	5	-	-	-	2	6*	-							
	6	-	-	-	2	8*	-							

Here, m = No. of rows = 3

n = No. of columns = 6

m + n - 1 = 3 + 6 - 1 = 8

Also no. of allocated cells = 8 = m + n - 1.

So the solution is nondegenerate.

Table : 2 Showing Basic Feasible Solution (Non Optimal)

Plant \ Warehouse	W$_1$	W$_2$	W$_3$	W$_4$	W$_5$	Dummy	Plant Capacity	Row Nos. (U$_j$)
F$_1$	74	(280) 56	54	(120) (-) 62	68	(+) 0	400	u$_1$ = 8
F$_2$	(200) 58	64	62	58	(200) (+) 54	(100) (-) 74	500	u$_2$ = 0 (left)
F$_3$	66	70	(240) 52	(240) (+) 60	(120) (-) 60	0	600	u$_3$ = 6
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. (V$_j$)	V$_1$ = 58	V$_2$ = 48	V$_3$ = 46	V$_4$ = 54	V$_5$ = 54	V$_6$ = 0		

Calculation of Opportunity Costs for Basic Feasible Solution

Unallocated Cell	Opportunity Cost [$\Delta_{ij} = C_{ij} - (u_i + v_j)$]
F$_1$ - W$_1$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 74 - (8 + 58) = 8$
F$_1$ - W$_3$	$\Delta_{13} = C_{13} - (u_1 + v_3) = 54 - (8 + 46) = 0$
F$_1$ - W$_5$	$\Delta_{15} = C_{15} - (u_1 + v_5) = 68 - (8 + 54) = 6$

F_1 - Dummy	$\Delta_{16} = C_{16} - (u_1 + v_6) = 0 - (8 + 0) = -8$
F_2 - W_2	$\Delta_{22} = C_{22} - (u_2 + v_2) = 64 - (0 + 48) = 6$
F_2 - W_3	$\Delta_{23} = C_{23} - (u_2 + v_3) = 62 - (0 + 46) = 16$
F_2 - W_4	$\Delta_{24} = C_{24} - (u_2 + v_4) = 58 - (0 + 54) = 4$
F_3 - W_1	$\Delta_{31} = C_{31} - (u_3 + v_1) = 66 - (6 + 58) = 2$
F_3 - W_2	$\Delta_{32} = C_{32} - (u_3 + v_2) = 70 - (6 + 48) = 16$
F_3 - Dummy	$\Delta_{36} = C_{36} - (u_3 + v_6) = 0 - (6 + 0) = -6$

As all the Opportunity Costs are not nonnegative, the solution is non optimal i.e. further improvement is possible.

For this a loop is formed starting from the cell having highest negative value which is cell (F1 - Dummy) having a highest negative opportunity cost value of -8. The starting cell of the loop is marked with a (+) and thereafter alternately the corner cells of the loop are marked (–) and (+). Next the minimum of the allocated quantities of the cells marked (–) is subtracted from the allocated quantities of all the cells marked (–) and added to all the cells marked (+). This leads to an improved solution as shown below.

Table : 3 Showing Improved Solution (Optimal)

Warehouse / Plant	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity	Row Nos. [U]
F_1	74; 8	56; (280)	54; 0 (+)	62; (20) (-)	68; 6	0; (100)	400	$U_1 = 0$ (Let)
F_2	58; (200)	64; 16	62; 16	58; 4	54; (300)	0; 8	500	$U_2 = -8$
F_3	66; 2	70; 16	52; (240) (-)	60; (340) (+)	60; (20)	0; 2	600	$U_3 = -2$
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. (V_j)	$V_1 = 66$	$V_2 = 56$	$V_3 = 54$	$V_4 = 62$	$V_5 = 62$	$V_6 = 0$		

Opportunity Costs (Dij) for the unallocated cells are calculated same as before and shown in left bottom corner of the cells.

(a) As Dij ≥ 0, the solution is optimal.

Table -4: Showing Optimal Distribution Plan

From Plant	To Warehouse	Quantity (Units)	Cost/Unit (Rs.)	Total (Rs.)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
F_1	W_2	280	56	15680
	W_4	20	62	1240
	Dummy	100	0	0

F_2	W_1	200	58	11600
	W_5	300	54	16200
F_3	W_3	240	52	12480
	W_4	340	60	20400
	W_5	20	60	1200
Total		1500	–	Rs. 78800

Minimum Cost of Transportation is Rs. 78800

(b) Plant F1 is having a surplus quantity of 100 units.

(c) Presence of zero opportunity cost (in the cell F1 -W3) indicates that alternative optimum solution is possible for the problem. To get the solution, we form a loop starting from the cell F1 - W3. The new solution is shown below–

Table-5: Showing Alternative Optimum Solution

Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity
F_1	74	56 (280)	54 (20)	62	68	0 (100)	400
F_2	58 (200)	64	62	58	54 (300)	0	500
F_3	66	70	52 (220)	60 (360)	60 (20)	0	600
Warehouse Demand	200	280	240	360	320	100	1500

Table-6: Showing Alternative Optimum Distribution Plan

From Plant	To Warehouse	Quantity (Units)	Cost/Unit (Rs.)	Total (Rs.)
(1)	**(2)**	**(3)**	**(4)**	**(5) = (3) × (4)**
F_1	W_2	280	56	15680
	W_3	20	54	1080
	Dummy	100	0	0
F_2	W_1	200	58	11600
	W_5	300	54	16200
F_3	W_3	220	52	11440
	W_4	360	60	21600
	W_5	20	60	1200
Total		**1500**	**–**	**Rs. 78800**

So the alternative solution is given above.

Answer for Q.NO.21.

Initially in this problem there are four sources (factories) and three destinations (sales Depot) .

Total Cost/unit = Production cost/unit + Raw material cost/unit + Transportation Cost/unit.

Profit/unit = Selling Price/unit - (Total Cost/unit)

Total Availability = 310 units & Total requirement = 350 units Since Total Availability not equal to total requirement so it is a unbalanced transportation problem.

Since total availability is less than total requirement we have to introduce a dummy factory with adjustment of 40 units to make balance transportation problem.

Table showing the calculation of per unit Profit matrix

	Sales depot 1	Sales depot 2	Sales depot 3	Availability
Factory 1	34 - (15 + 10 + 3) = 6	32 - (15 + 10 + 1) = 6	1	10
Factory 2	-2	-2	-4	150
Factory 3	3	2	2	50
Factory 4	7	5	3	100
Factory 5 (Dummy)	0	0	0	40
Requirement	80	120	150	350

Table showing the Calculation of per unit cost matrix [Subtracting each element of cost from biggest element here it is '7']

	Sales depot 1	Sales depot 2	Sales depot 3	Availability
Factory 1	1	1	6	10
Factory 2	9	9	11	150
Factory 3	4	5	5	50
Factory 4	0	2	4	100
Factory 5 (Dummy)	7	7	7	40
Requirement	80	120	150	350

Now we can apply VAM to get initial Basic feasible solutions(IBFS)

For optimality solution we will follow two steps (1) Calculation of Row Penalty (ui) and column penalty (vj) by trial and error method on the basis of occupied solution (IBFS) .

Total no of initial basic feasible solutions = m+n-1 but total no of u_i and v_j are m + n so with the help of m+n-1 IBFS we can never solve m+n unknowns so any one of m+n solutions can be solved by trial and error method .

Any one of the u_i or v_j will be zero on the basis of maximum number of occupied cell if no of occupied cells are same for more than one rows or one columns we can consider any of them to maintain the condition of m+n-1.

Let C_{ij} be the cost for occupied cell .Using occupied cell costs and one of the trial solution we can calculate the other row penalties and column penalties.

Where, $C_{ij} = u_i + v_j$

After getting all ui and vj then we calculate the unoccupied cell using the formula given below:

C_{ij} - ($u_i + v_j$)

where, C_{ij} is the cost of unoccupied cell.

Answer for Q.NO.22.

Profit matrix:

	A	B	C	D	E	F	
W	275	350	425	225	150	0	300
X	300	325	450	175	100	0	250
Y	250	350	475	200	125	0	150
Z	325	275	400	250	175	0	200
	150	100	75	250	200	125	

Loss Matrix:

200	125	50	250	325	475	300/275/225/25
	25		50	200	25	
175	150	25	300	375	475	250/100/0
150					100	25/25/125/75/5
225	125	0	275	350	475	150/75/0
	75	75				125* 100*
150	200	75	225	300	475	
			200			200/0

75/50/50/75/75/75*

m + n – 1 allocation s are there, optimality test can be performed.

150	100	75	250	200	125
0	25	0	50	0	100
	0		0		0
25	0	25	25	25	0
25	0		25	25	0
25	25		25	25	0
25			25	25	0
			25	25	0
			50	50	0

M + n – 1 allocation s are there, optimality test can be performed.

200 / 25	125 / 25	50 / 50	250 / 50	325 / 200	475 / 25	0
175 / 150	150 / 25	25 / 25	300 / 50	375 / 50	475 / 100	0
225 / 50	125 / 75	0 / 75	275 / 25	350 / 25	475 / 0	0
150 / 0	200 / 100	75 / 100	225 / 200	300 / 0	475 / 25	-25
175	125	0	250	325	475	

As Δij ≥ 0, maximum profit is as follows.

Qty Maximum Profit

W →	B	25 × 350	=	8750
	D	50 × 225	=	11250
	E	200 × 150	=	30000
	F	25 × 0	=	0
X →	A	150 × 300	=	45000
	F	100 × 0	=	0
Y →	B	75 × 350	=	26250
	C	75 × 475	=	35625
Z →	D	200 × 250	=	50000
Max. Profit.		**900**		**Rs. 2,06,875**

Answer for Q.NO.23.

7	3	6	0	5/0	3	3*	-	-	-
4 / 5	6 / 5 3	8	0 / 2	10/8/3/0	4*	2	2*	2	2
5	8	4 / 7	0	7/0	4	1	1	4	-
8	4	3 / 3	0	3/0	3	1	1	1	1
5	8	10	2						

0	3	3	0
	0	0	
1	1	1	0
1	1	1	-
1	2	1	
-	2	1	-
-	2	5	-

	W	X	Y	Z	Ui
A	7 / 6	3 / 5	6 / 4	0 / 3	-3
B	4 / 5	6 / 3	8 / 3	0 / 2	0
C	5 / 2	8 / 3	4 / 7	0 / 1	-1
D	8 / 6	4	3 / 3	0 / 1	-2

	W	X	Y	Z	Ui
A	7 / 6	3 / 5	6 / 4	0 / 3	-3
B	4 / 5	6 / 3	8 / 3	0 / 2	0
C	5 / 2	8 / 3	4 / 7	0 / 1	-1
D	8 / 6	4	3 / 3	0 / 1	-2
Vj	4	6	5	0	

As Δij ≥ 0, the solution is optimum.

Allocation:

Minimum Cost

A → X → 5 × 3	=	15
B → W → 5 × 4	=	20
→ X → 3 × 6	=	18
→ Z 2 × 0	=	0
C → Y → 7 × 4	=	28
D → Y → 3 × 3	=	9
25		**Rs. 90**

Answer for Q.NO.24.

Profit matrix

6	-2	3	80
6	-2	2	110
1	-4	2	150
0	0	0	40
150	100	130	380

Loss Matrix:

0 (40)	8 (40)	3	80/40/0	3/3/5
0 (110)	8	4	110/2	4*
5	10 (20)	4 (130)	150/20/0	1/1/6*
6	6 (40)	6	10/0	0/0/0

150	100	130
40	0	0
0		
0	2	1
5*	2	1
	2	1

			U
0 (40)	8 (40)	3 / 1	0
0 (110)	8 / 0	4 / 2	0
5 / 3	10 (20)	4 (130)	2
6 / 4	6 (40)	6 / 6	-2
0	8	2	

As there are m+n-1 allocations, optimality test can be performed since Δij ≥ 0,

		Quantity	Maximum Profit
F1	W1	40 × 6	240
	W2	40 × -2	-80
F2	W1	110 × 6	660
F3	W2	20 × -4	-80
	W3	130 × 2	260
F4 Dummy	W2	40 × 0	0
		380	Rs. **1000**

Profit Rs. **1,000/-**

Answer for Q.NO.25.

Man \ Job	1	2	3	4	5	6
1	2	9	2	7	9	1
2	6	8	7	6	14	1
3	4	6	5	3	8	1
4	4	2	7	3	10	1
5	5	3	9	5	12	1
6	9	8	12	13	9	1

Row Operation* (Table - 1)

Man \ Job	1	2	3	4	5	6
1	1	8	1	6	8	0
2	5	7	6	5	13	0
3	3	5	4	2	7	0
4	3	1	6	2	9	0
5	4	2	8	4	11	0
6	8	7	11	12	8	0

* Matrix is obtained by subtracting min. element of each row of the given Matrix from all the elements of the corresponding row.

Column Operation* (Table - 2)

Man \ Job	1	2	3	4	5	6
1	0	7	0	4	1	0
2	4	6	5	3	6	0
3	2	4	3	0	0	0
4	2	0	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

* Matrix is obtained by subtracting min. element of each column of Table - 1 from all the elements of the corresponding column.

Table - 3

Man \ Job	1	2	3	4	5	6
1	$\boxed{0}$	7	0	4	1	0
2	4	6	5	3	6	$\boxed{0}$
3	2	4	3	$\boxed{0}$	0	0
4	2	$\boxed{0}$	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

All the zeros obtained in Table - 2 are covered by minimum no. of horizontal and vertical straight lines and shown above. Here order of the given matrix = 6 and minimum no. of horizontal and vertical lines = 4.

As 4 ≠ 6, the solution is non optimal.

Table - 4

Man \ Job	1	2	3	4	5	6
1	$\boxed{0}$	7	0	4	1	1
2	3	5	4	2	5	$\boxed{0}$
3	2	4	3	$\boxed{0}$	0	1
4	2	$\boxed{0}$	5	0	2	1
5	2	0	6	1	3	0
6	6	5	9	9	0	0

Above table is obtained by subtracting minimum uncovered element of Table - 3 from all the uncovered elements and by adding the same to all the elements at the junction of the intersecting straight lines.

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ 6 (order of the matrix). So the solution is non optimal.

Table - 5

Man \ Job	1	2	3	4	5	6
1	0	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\boxed{0}$	4	1	$\boxed{0}$	0	1
4	0	0	3	0	2	1
5	0	$\boxed{0}$	4	1	3	0
6	4	5	7	9	$\boxed{0}$	0

Above table is obtained by subtracting minimum uncovered element (2) of Table - 4 from all the uncovered elements and by adding the same to all the elements at the junction of the intersecting straight lines. Here minimum no. of horizontal or vertical straight lines to cover all the zeros = 6 = Order of the Matrix. So the solution is optimal.

Table - 6 Showing Optimum Solution - 1

Man \ Job	1	2	3	4	5	6
1	$\cancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\boxed{0}$	4	1	$\cancel{0}$	$\cancel{0}$	1
4	$\cancel{0}$	$\cancel{0}$	3	$\boxed{0}$	2	1
5	$\cancel{0}$	$\boxed{0}$	4	1	3	$\cancel{0}$
6	4	5	7	9	$\boxed{0}$	$\cancel{0}$

Table - 7 Showing Optimum Solution - 2

Man \ Job	1	2	3	4	5	6
1	$\cancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\cancel{0}$	4	1	$\boxed{0}$	$\cancel{0}$	1
4	$\boxed{0}$	$\cancel{0}$	3	$\cancel{0}$	2	1
5	$\cancel{0}$	$\boxed{0}$	4	1	3	$\cancel{0}$
6	4	5	7	9	$\boxed{0}$	$\cancel{0}$

Table - 8 Showing Optimum Solution - 3

Man \ Job	1	2	3	4	5	6
1	$\cancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\cancel{0}$	4	1	$\boxed{0}$	$\cancel{0}$	1
4	$\cancel{0}$	$\boxed{0}$	3	$\cancel{0}$	2	1
5	$\boxed{0}$	$\cancel{0}$	4	1	3	$\cancel{0}$
6	4	5	7	9	$\boxed{0}$	$\cancel{0}$

So the Optimal Assignments are as follows :—

As per Table - 6			As per Table - 7			As per Table - 8		
Man	Job	Time (hrs.)	Man	Job	Time (hrs.)	Man	Job	Time (hrs.)

1	3	2	1	3	2	1	3	2
2	6	1	2	6	1	2	6	1
3	1	4	3	4	3	3	4	3
4	4	3	4	1	4	4	2	3
5	2	3	5	2	3	5	1	5
6	5	9	6	5	9	6	5	9
Total	–	22	**Total**	–	22	**Total**	–	22

Minimum total operation time = 22 hrs.

Answer for Q.NO.26.

This is a problem of Maximisation. To solve it using Assignment technique it has to be converted to a Minimisation problem by forming a Relative Loss Matrix.

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	58	60	59	55	53

Relative Loss Matrix*

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	20	20	25	35	10
B	18	30	44	35	33
C	10	12	20	0	10
D	40	41	40	42	35
E	2	0	1	5	7

* This matrix is formed by subtracting all the elements of the given matrixfrom the highest element (60) of it.

Row Operation Matrix

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	10	10	15	25	0
B	0	12	26	17	15
C	10	12	20	0	10
D	5	6	5	7	0
E	2	0	1	5	7

Column Operation Matrix

Batting Position

Batsman \ Batting Position	III	IV	V	VI	VII
A	10	10	14	25	$\boxed{0}$
B	$\boxed{0}$	12	25	17	15
C	10	12	19	$\boxed{0}$	10
D	5	$\boxed{6}$	4	7	0
E	2	0	0	5	7

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order of the matrix(5). So the solution is non optimal.

Improved Matrix

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	10	6	10	25	$\boxed{0}$
B	$\boxed{0}$	8	21	17	15
C	10	8	15	$\boxed{0}$	10
D	5	2	$\boxed{0}$	7	$\cancel{0}$
E	6	$\boxed{0}$	$\cancel{0}$	9	11

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix.

So the solution is optimal.

Optimal Assignment

Batsman	Batting Position	Average runs scored
A	VII	50
B	III	42
C	VI	60
D	V	20
E	IV	60
	Total =	**232**

Expected maximum total runs = 232

Answer for Q.NO.27.

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

(a) Matrix after Row Operation

Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	2	3	2	1	4	5
02	4	4	6	3	2	5
03	6	10	8	4	7	6
04	8	7	6	5	3	9
05	7	3	4	5	4	3
06	5	5	6	7	8	1

Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	1	2	1	0	3	4
02	2	2	4	1	0	3
03	2	6	4	0	3	2
04	5	4	3	2	0	6
05	4	0	1	2	1	0
06	4	4	5	6	7	0

To find out the allocation of the Old Machines to the operators we consider the given matrix without the new machine.

Matrix after Column Operation

Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	0	2	0	0	3	4
02	1	2	3	1	0	3
03	1	6	3	0	3	2
04	4	4	2	2	0	6
05	3	0	0	2	1	0
06	3	4	4	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ order of the matrix (6). So the solution is non optimal.

Improved matrix

Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	0	2	0	1	4	5
02	0	1	2	1	0	3
03	0	5	2	$\boxed{0}$	3	2
04	3	3	1	2	0	6
05	3	0	0	3	2	1
06	2	3	3	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

Optimal Assignment

Operators	01	→	M3	-	2
	02	→	M1	-	4

	03	→	M4	-	4
	04	→	M5	-	3
	05	→	M2	-	3
	06	→	M6	-	1

17 Hours Minimum Operation Time

(b) & (c)

Operator	**Machines**					
	M1	**M2**	**M3**	**M4**	**M5**	**M6**	**New**
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6
Dummy	0	0	0	0	0	0	0

With the introduction of a new machine into the system, the problem becomes unbalanced one. To make it balanced, a Dummy operator is introduced and all the elements of the matrix coresponding to it are taken as zero.

(1) Matrix after Row Operation

Operator	**Machines**						
	M_1	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**	**New**
01	1	2	1	0	3	4	5
02	3	3	5	2	1	4	0
03	5	9	7	3	6	5	0
04	5	4	3	2	0	6	1
05	~~4~~	~~0~~	~~1~~	~~2~~	~~1~~	~~0~~	~~9~~
06	4	4	5	6	7	0	5
Dummy	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

As all the columns contain zeros, the matrix after column operation will remain same. So the operation need not be done.

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix(7). So the solution is non optimal.

(2) Improved Matrix

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	0	1	0	0	3	4	5
02	2	2	4	2	1	4	0
03	4	8	6	3	6	5	0
04	4	3	2	2	0	6	1
05	4	0	1	3	2	1	10
06	3	3	4	6	7	0	5
Dummy	0	0	0	1	1	1	1

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix(7). So the solution is non optimal.

(3) Improved Matrix

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	0	2	0	0	4	5	6
02	1	2	3	1	1	4	0
03	3	8	5	2	6	5	0
04	3	3	1	1	0	6	1
05	3	0	0	2	2	1	10
06	2	3	3	5	7	0	5
Dummy	0	1	0	1	2	2	2

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix(7). So the solution is non optimal.

(4) Improved Matrix Showing Optimal Solution (i)

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	$\boxed{0}$	2	0	0	5	6	7
02	0	1	2	$\boxed{0}$	1	4	0
03	2	7	4	1	6	5	$\boxed{0}$
04	2	2	0	0	$\boxed{0}$	6	1
05	3	$\boxed{0}$	0	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	0	1	$\boxed{0}$	1	3	3	3

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 7 = order of the matrix. So the solution optimal.

Improved Matrix Showing Optimal Solution (ii)

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	$\cancel{0}$	2	$\boxed{0}$	$\cancel{0}$	5	6	7
02	$\cancel{0}$	1	2	$\boxed{0}$	1	4	$\cancel{0}$
03	2	7	$\cancel{0}$	1	6	5	$\boxed{0}$
04	2	2	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	6	1
05	3	$\boxed{0}$	0	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	$\boxed{0}$	1	$\cancel{0}$	1	3	3	3

Improved Matrix Showing Optimal Solution (iii)

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	$\cancel{0}$	2	$\cancel{0}$	$\boxed{0}$	5	6	7
02	$\boxed{0}$	1	2	$\cancel{0}$	1	4	$\cancel{0}$
03	2	7	4	1	6	5	$\boxed{0}$
04	2	2	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	6	1
05	3	$\boxed{0}$	$\cancel{0}$	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	$\cancel{0}$	1	$\boxed{0}$	1	3	3	3

Table Showing Multiple Optimum Allocations

Solution (i)			Solution (ii)			Solution (iii)		
Operators	**M/C**	**Time (Hrs.)**	**Operators**	**M/C**	**Time (Hrs.)**	**Operators**	**M/C**	**Time (Hrs.)**
01	M1	2	01	M3	5	01	M4	1
02	M4	3	02	M4	1	02	M1	4
03	New	1	03	New	6	03	New	1
04	M5	3	04	M5	0	04	M5	3
05	M2	3	05	M2	3	05	M2	3
06	M6	1	06	M6	7	06	M6	1
Dummy	M3	0	Dummy	M1	3	Dummy	M3	0
Total	–	13*	**Total**	–	13*	**Total**	–	13*

***** Minimum Operation Time

From above it can be said that replacement of an old machine with the new one will result in a reduction in Total Operating Time by 17-13 = 4 Hours. So replacement decision is advantageous.

As per solutions (i) & (iii) above, Machine M3 should be replaced by a New Machine and as per Solution (iii), M_1 should be replaced by a New one.

Answer for Q.NO.28.

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	40	5	45	20	15	20
C	25	60	10	65	25	10
D	25	20	35	10	25	60
E	30	70	40	5	40	50
F	10	25	30	40	50	15

Matrix after Row Operation

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	35	0	40	15	10	15
C	15	50	0	55	15	0
D	15	10	25	0	15	50
E	25	65	35	0	35	45
F	0	15	20	30	40	5

Matrix after Column Operation

Salesman	Region					
	I	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	35	0	40	15	0	15
C	15	50	0	55	5	0
D	15	10	25	0	5	50
E	25	65	35	0	25	45
F	0	15	20	30	30	5

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

Improved Matrix (Optimal)

Salesman	Region					
	I	**II**	**III**	**IV**	**V**	**VI**
A	20	35	$\boxed{0}$	30	~~0~~	45
B	40	$\boxed{0}$	40	20	~~0~~	15
C	20	50	~~0~~	60	5	$\boxed{0}$
D	15	5	20	~~0~~	$\boxed{0}$	45
E	25	60	30	$\boxed{0}$	20	40
F	$\boxed{0}$	10	15	30	30	~~0~~

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

Table showing optimal allocation

Salesman	Region	Cost (Rs.)	
A	III	0	
B	II	5	
C	VI	10	
D	V	25	
E	IV	5	
F	I	10	
Total		**Rs. 55**	Minimum Cost

(b) The given problem is a problem of Maximisation. To convert it to a problem of Minimisation, a Relative Loss Matrix is formed by subtracting all the elements of the given matrix from the highest element (70).

Relative Loss Matrix Matrix after Row Operation

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	55	35	70	45	60	25
B	30	65	25	50	55	50
C	45	10	60	5	45	60
D	45	50	35	60	45	10
E	40	0	30	65	30	20
F	60	45	40	30	20	55

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	30	10	45	20	35	0
B	5	40	0	25	30	25
C	40	5	55	0	40	55
D	35	40	25	50	35	0
E	40	0	30	65	30	20
F	40	25	20	10	0	35

Matrix after Column Operation

Salesman	Region					
	I	**II**	**III**	**IV**	**V**	**VI**
A	25	10	45	20	35	0
B	0	40	0	25	30	25
C	35	5	55	0	40	55
D	30	40	25	50	35	0
E	35	0	30	65	30	20
F	35	25	20	10	0	35

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

Improved Matrix

Salesman	Region					
	I	**II**	**III**	**IV**	**V**	**VI**
A	5	10	25	20	35	0
B	0	60	0	15	50	45
C	15	5	35	0	40	55
D	10	40	5	50	35	0
E	15	0	10	65	30	20
F	15	25	0	10	0	35

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is optimal.

Improved Matrix (Optimal)

Salesman	Region					
	I	**II**	**III**	**IV**	**V**	**VI**
A	$\boxed{0}$	10	20	20	30	$\cancel{0}$
B	$\cancel{0}$	65	$\boxed{0}$	50	50	50
C	10	5	30	$\boxed{0}$	35	55
D	5	40	$\cancel{0}$	50	30	$\boxed{0}$
E	10	$\boxed{0}$	5	65	25	20
F	15	30	$\cancel{0}$	15	$\boxed{0}$	40

Table Showing Optimal Allocation

Salesman	Region	Earning (₹)	
A	I	15	
B	III	45	
C	IV	65	
D	VI	60	
E	II	70	
F	V	50	
Total		**₹ 305**	Maximum Earning

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

(c) The cost matrix after imposing the given restriction is

Region

		I	II	III	IV	V	VI
Sales man	A	15	35	0	25	α	45

	B	40	5	45	20	15	10
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	α	40	5	40	50
	F	10	25	30	40	50	15

Cost (figures are in Rs.)

(Whenever such restrictions are imposed, we have to consider the corresponding element of the given matrix as infinitely large i.e. a)

<u>Answer for Q.NO.29.</u>

Relative Loss Matrix

Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	12	17	22	27
III	12	17	22	27
IV	18	22	26	30

As this is a problem of Maximisation, the same is converted to one of Minimisation by firming a Relative Loss Matrix where all the elements of the given matrix are subtracted from the highest element of the matrix (which is 42 in this case)

Matrix after Row Operation

Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	0	5	10	15
III	0	5	10	15
IV	0	4	8	12

Matrix after Column Operation

Jobs \ M/cs	A	B	C	D
I	0	3	6	9
II	0	1	2	3
III	0	1	2	3
IV	0	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 2 ≠ Order of the matrix (4)

So the solution is non optimal.

Improved Matrix (Non Optimal)

Jobs \ M/cs	A	B	C	D
I	0	2	5	8
II	0	0	1	2
III	0	0	1	2
IV	1	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 3 ≠ Order of the matrix (4)

So the solution is non optimal.

Further Improved Matrix [Optimal Solution (i)]

Jobs \ M/cs	A	B	C	D
I	0	2	4	7
II	0	0	0	1
III	0	0	0	1
IV	2	1	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 = Order of the matrix. So the solution is optimal.

Further Improved Matrix (Optimal Solution-ii)

Jobs \ M/cs	A	B	C	D
I	$\boxed{0}$	2	4	7
II	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
III	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
IV	2	1	$\cancel{0}$	$\boxed{0}$

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 = Order of the matrix.

So the solution is optimal.

Further Improved Matrix (Optimal Solution-ii)

Jobs \ M/cs	A	B	C	D
I	$\boxed{0}$	2	4	7
II	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
III	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
IV	2	1	$\cancel{0}$	$\boxed{0}$

Assignment as per Soution (i)			Assignment as per Soution (ii)		
Jobs	M/cs	Profit (₹)	Jobs	M/cs	Profit (₹)
I	A	42	I	A	42
II	B	25	II	C	20
III	C	20	III	B	25
IV	D	12	IV	D	12
Total	–	₹ 99	Total	–	₹ 99

Answer for Q.NO.30.

From \ To	A	B	C	D	E
A	-	12	24	25	15
B	6	-	16	18	7
C	10	11	-	18	12
D	14	17	22	-	16
E	12	13	23	25	–

Row Operation*

(Table - 1)

From \ To	A	B	C	D	E
A	-	0	12	13	3

* This matrix is obtained by subtracting minimum element of each row of the given

B	0	-	10	12	1	matrix from all the elements of the corresponding row.
C	0	1	-	8	2	
D	0	3	8	-	2	
E	0	1	11	13	-	

Column Operation*

(Table - 2)

To / From	A	B	C	D	E
A	–	$\boxed{0}$	4	5	2
B	0	–	2	4	$\boxed{0}$
C	0	1	–	$\boxed{0}$	1
D	0	3	$\boxed{0}$	–	1
E	$\boxed{0}$	1	3	5	–

* This matrix is obtained by subtracting minimum element of each column of Table-1 from all the elements of the corresponding column.

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix.

So the solution is optimal.

Now the solution obtained from the above table shows the travel route of the salesman as A to B, B to E, E to A which means the person is not visiting C and D at all while travelling back to A.

But this is not allowed as per the question.

So the matrix of Table-2 is examined for some of the next best solution which is depicted below.

To / From	A	B	C	D	E
A	–	$\boxed{0}$	4	5	2
B	0	–	2	4	$\boxed{0}$
C	$\boxed{0}$	1	–	0	1
D	0	3	$\boxed{0}$	–	1
E	0	1	3	$\boxed{5}$	–

Here the assignments have been started by encircling only zero present in the first row which means initial travel route A to B.

Then the only zero present in the last column is encircled which shows subsequent route B to E. Next the only zero of the last row is not encircled because in that case the route would have been E to A which is restricted as per the given condition. So that element of this row is considered which satisfies the restriction. It is 5 indicating the route as E to D. Next the only zero of 3rd column is encircled which means the route as D to C. Finally the only zero row present in the 3rd row is encircled which shows the route as C to A.

Hence the complete route of the Salesman is : A → B → E → D → C → A

Total distance travelled = 12 + 7 + 25 + 22 + 10 = 76 Kms.

This is the optimum distance.

Answer for Q.NO.31.

(a) First arrange the jobs as per the shortest processing time (SPT) sequence.

Job (j)	2	3	5	4	1
Processing time (tj) hrs	8	10	16	28	30

Therefore, the job sequence that minimises the mean flow time is 2-3-5-4-1.

Computation of minimum flow time (F min)

The flow time is the amount of time the job 'j' spends in the system. It is a measure which indicates the waiting of jobs in the system. It is the difference between the completion time (C_j) and ready time (R_j) for job j.

$F_j = C_j - R_j$

Job (j)	2	3	5	4	1
Processing time (t_j) hrs	8	10	16	28	30
Completion time (C_j)	8	18	34	62	92

Since the ready time (Rj) = 0 for all j, the flow time j ü is equal to Cj for all j.

Mean flow time = $\left(\bar{F}\right)=\frac{1}{n}\sum_{j=1}^{n}F_j=\frac{1}{5}[8+18+34+62+92]=\frac{1}{5}[214]=42.8\text{hours}$

(b) The weights are given as follows:

Job (j)	1	2	3	4	5
Processing time (t_j) hrs	30	8	10	28	16
Weight (W_j)	1	2	1	2	3

The weighted processing time = $\frac{\text{Processing time}(t_j)}{\text{Weight}(W_j)}$

The weighted processing time is represented as

Job (j)	1	2	3	4	5
Processing time (t_j hrs)	30	8	10	28	16
Weight (W_j)	1	2	1	2	3
Weighted Processing time (tj/ W_j)	30	4	10	14	5.31

Thus, arranging the jobs in the increasing order of tj/Wj (weighted shortest processing time WSPT) we have

Job (j)	2	5	3	4	1
Weighted Processing line (t_j/W_j)	4	5.31	10	14	30

optimal sequence that minimises the weighted mean flow time is 2-5-3-4 -1.

Weighted Mean flow time $\left(\bar{F}_w\right)$: $\bar{F}_w=\frac{\sum_{j=1}^{n}W_jF_j}{\sum_{j=1}^{n}W_j}$

Job (j)	2	5	3	4	1
Processing time (t_j) hrs	8	16	10	28	30
$F_j = (C_j - R_j)$	8	24	34	62	92
W_j	2	3	1	2	1
F_j x W_j	16	72	34	124	92

The weighted mean flow time is computed as follows for optimal sequence.

Weighted mean flow time w ü is computed as

$$\bar{F}_w = \frac{(16+72+34+124+92)}{(2+3+1+2+1)} = 37.55\text{hrs.}$$

Answer for Q.NO.32.

a. The arrival rate is given in the problem: λ = 16 customers per hour. Change the service time to a comparable hourly rate by first restating the time in hours and then taking its reciprocal. Thus, (3 minutes per customer)/ (60 minutes per hour) = 1/20 = 1/ μ. Its reciprocal is μ = 20 customers per hour = Service Rate.

b. Average no. of customers being served at any time.

r = λ / μ

= 16 / 20 = 0.80 customer.

Formulas for basic single-server model

Performance Measure	Equation
Average number in line/queue	$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$
Probability of zero units in the system	$P_o = 1 - \left(\frac{\lambda}{\mu}\right)$
Probability of *n* units in the system	$P_n = P_o\left(\frac{\lambda}{\mu}\right)^n$
Probability of less than *n* units in the system	$P_{<n} = 1 - \left(\frac{\lambda}{\mu}\right)^n$

c. Given: L_q = 3.2 customers

$L_s = L_q + r$ = 3.2 + 0.80 = 4.0 customers

Average time customers wait in line

= $W_q + L_q / \lambda$ = 3.2 / 16 = 0.20 hour, or 0.20 hour × 60 minutes/hour = 12 minutes

W_s = Average time customers wait in system = Wq + 1 / μ

Waiting time in line plus service

0.20 + 1 / 20 hour, or 15 minutes

d. System utilization is ρ = λ / M × μ

For M = 1, ρ = 16 / 1(20) = 0.80

For M = 2, ρ = 16 / 2(20) = 0.40

For M = 3, ρ = 16 / 3(20) = 0.27

Note that as the system capacity is measured by Mμ increases, the system utilization for a given arrival rate decreases.

<u>Answer for Q.NO.33.</u>

Arrival Rate = λ = 15 customers per hour

Service Rate = μ = 1

Service Time = 1 customer

3 minutes × 60 minutes per hour = 20 customers per hour

a. System Utilisation = ρ = λ / Mμ = λ / 1(20) = 0.75

b. Percentage of time the server will be idle = 1 – r = 1– 0.75 = 0.25, or 25 percent

c. Expected no. of customers waiting to be served = Lq = λ / μ(μ – λ) = 225 / 20(20 – 15) = 225 / (20 × 5) = 225 / 100 = 2.25 customers

d. Average time customers will spend in the system = WS = Lq / λ + 1μ = 2.25 /15 + 1 / 20 = 0.20 hours, or 12 minutes

e. Probability of zero customer in the system = P_0 = 1 – λ / μ = 1 – 15 / 20 = 0.25 and Probability of 4 customers in the system P_4= P_0 = $(\lambda / \mu)^4$ = 0.25 $(15 / 20)^4$ = 0.079

<u>Answer for Q.NO.34.</u>

Arrive Rate = λ = 8 cars per hour

Service Rate = μ = 1 per 5 minutes, or 12 per hour

Av. no. of cars waiting in line = Lq = λ^2 / 2μ(μ – λ) = 8^2 / 2(12)(12 – 8) = 0.667 car

Av. time cars spend in line and service = W_S = L_q / λ + 1/μ = 0.667 / 8 + 1 / 12 = 0.167 hours, or 10 minutes

<u>Answer for Q.NO.35.</u>

The usual notations are given:

Arrival Rate = λ = 20 customers / hour and service rate = μ = 24 customers / hour

Average no. of customers in the system = L_q = λ / μ(μ – λ) = 20 / (24 – 20) = 20 / 5 = 5 customers

Average time a customer spends in the system = W_S = L_S / λ = 5 / 20 = 1/4 = 0.25 hour = 15 mins

<u>Answer for Q.NO.36.</u>

Arrival rate = λ = 2 per hour

Service rate = μ = 3 per hour

(i) Average number of machinists being served or waiting to be served at any given time:

L_S = λ / μ(μ – λ) = 2 / (3 – 2) = 2

(ii) Average Time a machinist spends waiting for the services:

W_q = λ / μ × 1 / (μ – λ) = 2 / 3 × 1 / (3 – 2) = 0.667 hours

It means a machinist spends 40 minutes (ie., 60 × 0.667) in the queue.

(iii) Average time in the system

$W_S = 1 / (\mu - \lambda) = 1 / (3 - 2) = 1$ hour

Average number of machinists in the system = 2 [As per (i) above]

Cost of two machinists being away from work = Rs. 4 × 2 = Rs.8.00 per hour

Attendant cost = 1.50 per hour / 9.50 per hour

Cost of 8- hour day = 8 hrs × Rs.9.50 = Rs.76.00

(iv) It is assumed that there is still a single service point, but the average service rate with 2 attendants now is 4 per hour

∴ Now $\lambda = 2$ per hour

m = 4 per hour

∴ Average number of machinists in the system = $L_S = \lambda / \mu - \lambda = 2 / (4 - 2) = 1$

Average time spent by a machinist in the system = WS = $1/ (\mu - \lambda) = 1 / (4 - 2) = 12$ hour

Machinists cost = ½ hr × Rs.4 =	Rs. 2.00
Attendant cost (@ 1.50 per attendant × 2 attendants	Rs. 3.00
Total Cost	Rs. 5.00

Cost per 8 – hour day = Rs.5 × 8 hrs. = Rs.40.00

Answer for Q.NO.37.

Here, Arrival Rate = $\lambda = 60 / 60$ per second = 1 per minute

Service Rate = $\mu = 60 / 60$ per second = 1.5 per minute

(i) Average queue length:

$L_q = \lambda\mu / \times \lambda / (\mu - \lambda) = 1 / 1.5 \times 1 / (1.5 - 1) = 1 / 0.75 = 4 / 3$ workers

(ii) Average length of non-empty queues:

$L_n = \mu / \mu - \lambda = 1 / (1.5 - 1) = 3$ workers

(iii) Average number of workers in the system:

$L_s = \lambda / \mu - \lambda = 1 / (1.5 - 1) = 2$ workers

(iv) Mean waiting time of an arrival

$W_q = \lambda / \mu \times \lambda / (\mu - \lambda) = 1 / 1.5 \times 1 / (1.5 - 1) = 34$ minutes

(v) Average waiting time of an arrival who waits

$W_n = 1 / \mu - \lambda = 1 / (1.5 - 1) = 2$ minutes

Answer for Q.NO.38.

Table - I

Random No. Range Table for demand				
Demand per week	**Frequency (f)**	**Probability (p = f ÷ Σf)**	**Cumulative Probability**	**Range† of Random Nos.**
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41

15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
	Σf = 50	**1.00**		

†As the given Random Nos. are of 2 digits, the ranges of Random Nos. has also been considered to have 2 digits only. Also the range of Random Nos. corresponds to cumulative probability values which lies between 0 & 1 and can be correlated as nos. between 00 and 99.

Table - II

Simulated Values for next 10 weeks		
Weeks	**Random Nos.**	**Demand**
1	35*	10*
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
Total	–	**120**

*From Table (I), Random No. 35 appears in the range of 26-41. Also the demand for this range is 10.

Average weekly demand = 120 / 10 = 12

Answer for Q.NO.39.

Random No. Range Table			
Demand	**Probability**	**Cumulative Probability**	**Random No. Range**
15	.05	.05	00-04
16	.08	.13	5-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
Total	**1.00**	**–**	**–**

Calculation of demand and profit for next 20 years					
Year	**Random Numbers**	**Expected demand**	**No. of books unsold if stock is**		
			16*	**17***	**18***
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-
4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-
Total			**2**	**7**	**18**

*Lookiong at the simulated demand pattern, these stock figures have been chosen to find out optimal course of action. Stock figures of 20 or more have not been considered because it is quite obvious that such figures will not give optimal course of action due to high losses for the unsold books.

Statement Showing Computation of Profit			
No. of Books order (n)	**No. of Books sold in 20 years (n × 20 - Books unsold)**	***Net Profit (Rs.)**	**Average Profit/Year (Profit ÷ 20)**
15	15 × 20 = 300	Rs. 6000	Rs. 300
16	16 × 20 – 2 = 318	Rs. 6300 (318 × 20) – 2 × 30	Rs. 315
17	(17 × 20) – 7 = 333	Rs. 6450 (333 × 20) -7 × 30	Rs. 322.5

18	(18 × 20) – 18	Rs. 6300 (342 × 20) – 18 × 30	Rs. 315

* Net Profit = No. of books sold × Rs. 20# – No. of books unsold × Rs. 30**

Selling price/book = Rs. 80, Cost/book = Rs. 60

Profit /book = 80 – 60 = Rs. 20

Selling price of any unsold book = Rs. 30

**Loss incurred/unsold book = Rs. 60 – Rs. 30 = Rs. 30

Since profit is maximum for 17 books order, the optimal policy is to order 17 books per year.

<u>Answer for Q.NO.40.</u>

Range of random numbers							
Receipt (Rs.)	**Probability**	**Cumulative probability**	**Range**	**Payments (Rs.)**	**Probability**	**Cumulative probability**	**Range**
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulation of Data for a period of 12 weeks					
Week	**Random No. for receipt**	**Expected Receipt (Rs.)**	**Random No. for payment**	**Expected Payment (Rs.)**	**Week end Balance (Rs.)**
Opening Balance					**8000**
1	03	3000	61	6000	5000 (8000 + 3000 – 6000)
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)

Estimated balance at the end of 12th week = Rs. (3,000)

Highest balance = Rs. 7,000

Average balance during the quarter = 45,000/12 = Rs. 3,750

Answer for Q.NO.41.

Simulation of data of an Automobile Production line			
Production/day	**Probability**	**Cumulative Probability**	**Random No. Range**
95	0.03	0.03	00-02
96	0.05	0.08	03-07
97	0.07	0.15	08-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91
104	0.05	0.97	92-96
105	0.03	1.00	97-99
	1.00		

Simulated data				
Day	**Random No.**	**Production**	**No.of cars waiting to be shipped**	**No. of empty space on the boat**
1	20	98	-	3
2	63	101	-	-
3	46	100	-	1
4	16	98	-	3
5	45	100	-	1
6	41	100	-	1
7	44	100	-	1
8	66	101	-	-
9	87	103	2	-
10	26	99	-	2
11	78	102	1	-
12	40	100	-	1
13	29	99	-	2
14	92	104	3	-

15	21	98	-	3
Total			6	18

Average no. of cars waiting to be shipped = 6/15 = 0.40 per day

Average no. of empty space on the boat = 18/15 = 1.2 per day

<u>Answer for Q.NO.42.</u>

Demand	Probability	Cumulative Probability	Random No. Range
0	0.05	0.05	00-04
1	0.10	0.15	05-14
2	0.30	0.45	15-44
3	0.45	0.90	45-89
4	0.10	1.00	90-99

Option - A

Day	Random No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quantity for which Order Placed
1	89	3	8	-	5	-
2	34	2	5	6	9	-
3	70	3	9	-	6	0
4	63	3	6	-	3	5
5	61	3	3	0	0	-
6	81	3	0	5	2	5
7	39	2	2	-	0	5
8	16	2	0	5	3	-
9	13	1	3	5	7	-
10	73	3	7	-	4	5
					39	

Ordering cost 4 × 10	Rs. 40
Carrying cost 0.5 × 39	Rs. 19.50
Total Cost	Rs. 59.50

Option B

Day	R No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quantity for which Order placed
1	89	3	8	-	5	-

2	34	2	5	6	9	-
3	70	3	9	-	6	-
4	63	3	6	-	3	8
5	61	3	3	-	0	-
6	81	3	0	8	5	-
7	39	2	5	-	3	8
8	16	2	3	-	1	-
9	13	1	1	8	8	-
10	73	3	8	-	5	-
					45	

Ordering cost 2 × 10	Rs. 20.0
Carrying cost 0.5 × 45	Rs. 22.50
Total Cost	Rs. 42.50

Option 'B' is better because it has low Inventory cost.

Answer for Q.NO.43.

Inter-arrival time				Service time			
Minutes	**Probability**	**Cumulative probability**	**Range of Random No.**	**Minutes**	**Probability**	**Cumulative probability**	**Range**
2	0.22	0.22	00-21	4	0.28	0.28	00-27
4	0.30	0.52	22-51	6	0.40	0.68	28-67
6	0.24	0.76	52-75	8	0.22	0.90	68-89
8	0.14	0.90	76-89	10	0.10	1.00	90-99
10	0.10	1.00	90 - 99	–	–	–	–

Sl.No.	Random No. for inter arrival time	Inter arrival time (Mins.)	Entry time in queue as per clock	Service start time as per clock	Random no for service time	Service time (Mins.)	Service end time as per clock	Waiting time of customer (Mins.)	Idle time (Mins.)
1	78	8	8.08	8.08	44	6	8.14	-	8
2	26	4	8.12	8.14	21	4	8.18	2	-
3	94	10	8.22	8.22	73	8	8.30	-	4
4	08	2	8.24	8.30	96	10	8.40	6	-

5	46	4	8.28	8.40	63	6	8.46	12	-
6	63	6	8.34	8.46	35	6	8.52	12	-
7	18	2	8.36	8.52	57	6	8.58	16	-
8	35	4	8.40	8.58	31	6	9.04	18	-
9	59	6	8.46	9.04	84	8	9.12	18	-
10	12	2	8.48	9.12	24	4	9.16	34	-
11	97	10	8.58	9.16	05	4	9.20	18	-
12	82	8	9.06	9.20	37	6	9.26	14	-
Total Time								**140**	**12**

Average time spent by the customer waiting in the queue = 140/12 = 11.67 minutes

Probability of idle time of petrol station = Total idle time / Total Operating = 12/86 = 0.1395 time of the Service Channel*

*Service End Time – 9.26 Hrs. Service Channel opened at 8.00 hrs. i.e. Total Time of the Service Channel = 1 hr. 26 Mins = 86 Mins.

Answer for Q.NO.44.

Table-1: Probability Distribution (Supply)

Supply	Probability	Cum. Prob.	Range	Range of Random Nos. for simulation
10	40/500 = 0.08	0.08	0 - 0.08	00 - 07
20	50/500 = 0.10	0.18	0.08 - 0.18	08 - 17
30	190/500 = 0.38	0.56	0.18 - 0.56	18 - 55
40	150/500 = 0.30	0.86	0.56 - 0.86	56 - 85
50	70/500 = 0.14	1.00	0.86 - 1.00	86 - 99

Table-2: Probability distribution (Demand)

Demand	Probability	Cum. Prob.	Range	Range of Random Nos. for simulation
10	50/500 = 0.10	0.10	0 - 0.10	00 - 09
20	110/500 = 0.22	0.32	0.10 – 0.32	10 – 31
30	200/500 = 0.40	0.72	0.32 - 0.72	32 – 71
40	100/500 = 0.20	0.92	0.72 – 0.92	72 – 91
50	40/500 = 0.08	1.00	0.92 – 1.00	92 – 99

Table-3: Showing simulated data

Simulated data for supply			Simulated data for demand		
Day	**Random No.**	**Supply (Kg.)**	**Day**	**Random No.**	**Demand (Kg.)**
1	31	30	1	18	20
2	63	40	2	84	40
3	15	20	3	79	40
4	07	10	4	32	30

5	43	30	5	75	40
6	81	40	6	27	20

Table-4: Statement Showing Supply, Demand and Profit

Day	Supply	Demand	*Sales Revenue	Cost (II)	Loss due to unsatisfied demand (III)	Profit (Rs.)
(a)	(b)	(c)	(d)	(e) = (b) × Rs.20/kg	(f) = [(c)–(b)]× Rs.8/kg	(g) = (d)-(c)-(f)
1	30	20	600	600	-	Nil
2	40	40	1,200	800	-	400
3	20	40	600	400	160	40
4	10	30	300	200	160	-60**
5	30	40	900	600	80	220
6	40	20	600	800	-	-200**

* (1) Sales revenue = Demand × Selling price, when Demand < Supply

(2) Sales revenue = Supply × Selling price, when Demand > Supply

** Negative figures indicate loss

<u>Answer for Q.NO.45.</u>

The steps below correspond to those in Fig. 5-17.

(1) Data are given in frequencies.

(2) To formulate a probability distribution, divide each frequency by the total (60), for example, 6 60 = .10 and 18 60 = .30. Then formulate a cumulative probability distribution by successively summing the probability values.

Demand (tons/day)	Frequency (days)	Probability P(X)	Cumulative probability
10	6	0.10	0.10
11	18	0.30	(10 + 0.30) = 0.40
12	15	0.25	0.65
13	12	0.20	0.85
14	6	0.10	0.95
15	3	0.05	
1.00	60	1.00	

(3) Next, assign random – number intervals so that the number of values available to each class corresponds with the probability. Using 100 two – digit numbers (00-99), we assign 10 percent (00-09) to the first class, 30 percent (10-39) to the second class, and so on.

Demand (tons/day)	Probability P(X)	Corresponding Random Numbers
10	.10	00-09
11	.30	10-39
12	.25	40-64
13	.20	64-84
14	.10	85-94
15	.05	95-99
	1.00	

RN = 27

(4) We obtained random numbers (RN) from column 1 of Appendix I (for convenience), so the first seven numbers are:

27 13 80 10 54 60 49

The first RN, 27, falls into the second class of the distribution and corresponds to a demand of 11 tons per day.

Random Number	27	13	80	10	54	60	49
Simulated Demand	11	11	13	11	12	12	12

(5) This extremely small simulation yields a mean of X = 11.7 tons and a standard deviation of s = .76 tons. The expected value from the empirical probability distribution is E(X) = [XP(X)] = 12.05 tons, suggestion that the small sample size of only 7 periods has resulted in some error. A much larger sample should be simulated before the simulation results are used for making decisions.

Note that the width of the random number "target" in each class corresponds exactly to the relative frequency of the class. This helps to ensure that the simulated results have the same type of distribution as the original data. This is more apparent in the graphic method where the vertical distances on the graph correspond to the relative frequencies of the respective classes.

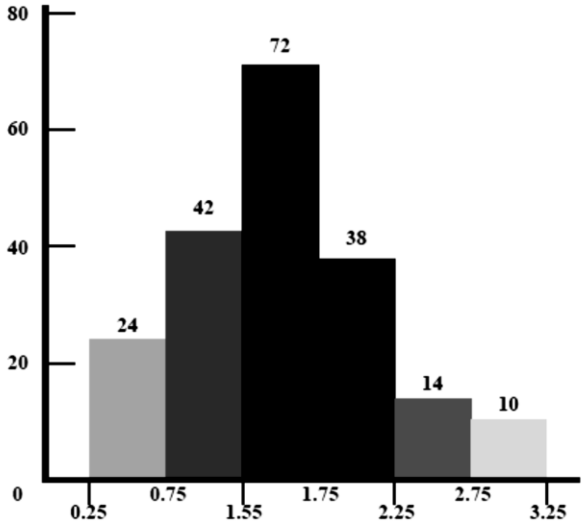
<u>Answer for Q.NO.46.</u>

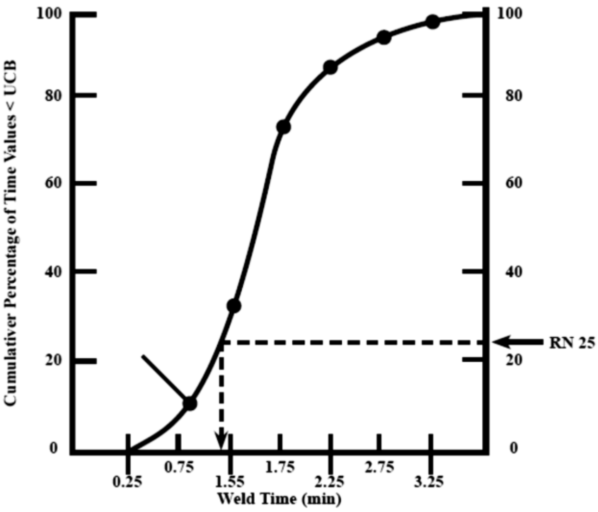
(a) Cumulative distributions are usually formulated on a scale where the cumulative percentage is "more than" or "less than" a corresponding X axis amount. We shall use a "less than" percentage and so will need to identify the upper- class boundaries (UCB) as the Y coordinates for the cumulative distribution.

Weld Time (Min)	Frequency In Numbers	Upper – Class Boundary (UCB)	Cumulative Number Of Times < UCB	Cumulative Percentage Of Time < UCB
< .25	0	.25	0	0

.25 < .75	24	.75	24	12
.75 < 1.25	42	1.25	66	33
1.25 < 1.75	72	1.75	138	69
1.75 < 2.25	38	2.25	176	88
2.25 < 2.75	14	2.75	190	95
2.75 < 3.25	10	3.25	200	100

(b) The frequency distribution is constructed by extending vertical lines from the class boundaries to the appropriate frequency level for the class. For the cumulative distribution, values of the cumulative percentage of time < UCB are plotted at weld times corresponding to the UCB. For example, the frequency (12 percent) is plotted at UCB = .75 (as illustrated below).





(c) The simulated time for random number (RN) 25 is determined by entering the cumulative graph at 25 (as shown by the arrow) and proceeding horizontally to the curve and then down to the weld time. The resultant is a reading of 1.0 minute (rounded to the nearest. 25 minutes). Times for random number 90 and 59 are 2.5 and 1.5 minutes, respectively. (A larger graph would lend more accuracy.)

(d) From the cumulative distribution, about 12 percent of the times exceed 2.0 minutes

Answer for Q.NO.47.

(a) Our interest lies in activity b, so we can set up a table (below) to show when parts arrive at B, how long it takes B, how long it takes B to work on them, and the resultant idle and waiting times:

Part Number	Part Available for Activity B at Time	Activity B Beginning Time	Activity B Ending Time	Activity B Idle Time	Waiting Time of Part	Number parts Waiting at B End time
1	-	0	.5	0	0	0
2	.8	.8	1.2		.3	0
3	1.0	1.2	1.6	0	.2	1
4	1.5	1.6	2.5	0	.1	1
5	2.1	2.5	2.9	0	.4	2
6	2.6	2.9	3.5*	0	.3	2
7	2.9				1.0 **	2
8	3.2					

* Total run time.

**Total waiting time.

Activity B begins at 0, and it takes .5 minute to complete the first part. B is then idle for .3 minute until part 2 arrives from A at .8 minutes. Part 2 takes .4 minute, so the ending time is .8 + .4 = 1.2 minutes. By this time part 3 has been waiting. 2 minute because it became available at .8 + .2 = 1.0 minute, but work could not be begun on it until 1.2 minutes. However, before activity B is finished on part 3 at 1.6 minutes, part 4 has arrived (at 1.0 + .5 = 1.5 minutes) and so one part is waiting. We continue systematically in this manner through part 6, noting that when it is finished at time were 3.5 minutes,there are two parts waiting, for their availability times were 2.9 minutes and 3.2 minutes, respectively.

(b) The average length of the waiting line (that is, average inventory) ahead of B can be expressed in equation form as follows:

Average inventory = Total waiting time / Total run time

= 1.0 assembly minute / 3.5 minutes = 0.29 assembly

(c) Average output per hour:

$$\frac{6\,\text{unit}}{3.5\,\text{minutes}}\left(\frac{60\,\text{min}}{\text{hr}}\right) = 102.9\text{units/hr} = 102.9 \text{ units/hr}$$

= 102.9 units/hr

Answer for Q.NO.48.

Computation of Random Interval for Processing Time

Process time Minutes	A1			A2		
	Pi	ΣPi	Range	Pi	ΣPi	Range
10	0.10	0.10	0-9	0.20	0.20	0-19
11	0.15	0.25	10-24	0.10	0.60	20-59
12	0.40	0.65	25-64	0.20	0.80	60-79
13	0.25	0.90	65-89	0.15	.095	80-94
14	0.10	1.00	90-99	0.05	1.00	95-99

Simulated date for 15 units

	Random No.	Process Time	Random No.	Process Time	Total
1	41	12	34	11	23
2	74	13	76	12	25
3	49	12	43	11	23
4	83	13	43	11	24
5	11	11	83	13	24
6	11	11	83	13	24
7	36	12	02	10	22
8	94	14	45	11	25
9	54	12	15	10	22
10	75	13	05	10	23
11	00	10	89	13	23
12	08	10	80	13	23
13	74	13	28	11	24
14	34	12	24	11	23
15	93	14	09	10	24
		182		167	349

Average Process time for

A1 = 182/15 = 12.13 Minutes

A2 = 167/15 = 11.13 Minutes

For product = 349/15 = 23.27 Minutes

Expected process time for the product = 23.27 minutes (12 .13 + 11.13)

Answer for Q.NO.49.

Cost (Rs.)	Probability	Cumulative Probability	Random Range	Cost (Rs.)	Probability	Cumulative Probability	Random Range
17000	0.1	0.1	00-09	19000	0.1	0.1	00.09

18000	0.1	0.2	10-19	20000	0.1	0.2	10-19
19000	0.4	0.6	20-59	21000	0.2	0.4	20-39
20000	0.2	0.8	60-79	22000	0.4	0.8	40-79
21000	0.2	1.0	80-99	23000	0.15	0.95	80-94
				24000	0.05	1.00	95-99

Month	Random No. for Cost	Cost (Rs.)	Random No. for Sales	Cost (Rs.)	Monthly Net Revenue (Rs.)
1	82	21000	39	21000	-
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	-
5	36	19000	71	22000	3000
6	92	21000	83	23000	2000
7	73	20000	19	20000	-
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000
10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000
					35000

Average = 35000/20 = Rs. **1750.**

<u>Answer for Q.NO.50.</u>

(a) FCFS (First come first served) : Since the jobs are assigned letters A to E as they arrived to the shop, the sequence according to FCFS priority rule is A B C D E

(b) EDD (Early due date job first) rule : Taking into account the number of days until due date, the sequence of jobs as per EDD rules is

Job	B	E	C	A	D
No. of days units/due date	3	6	7	8	9

Here the job having earliest due date is sequenced first and the others are sequenced in ascending order of due date.

(c) L.S. (Least slack) rule also called as Minimum slack rule.

Calculation of slack :

Slack = (Number of days until due date) - (Number of days of work remaining)

Job	No. of days until/due date	No. of days of work remaining	Slack (Days)
A	8	7	8 – 7 = 1
B	3	4	3 – 4 = -1
C	7	5	7 – 5 = 2
D	9	2	9 – 2 = 7
E	6	6	6 – 6 = 0

Sequence :

Job	B	E	A	C	D
Slack -	1	0	1	2	7

Here the jobs are sequenced in ascending order of magnitude of their respective slacks.

(d) SPT (Shortest Processing Time job first) also referred as SOT (Shortest Operation time job First) rule or MINPRT (Minimum Processing time job first) rule. As per this rule, jobs are sequenced in ascending order of magnitude of their respective processing time.

Sequence :

Job	D	B	C	E	A
Processing Time (Days)	2	4	5	6	7

(e) LPT (Longest Processing time job first) also referred to as LOT (Longest operation time job first) rule.

As per this rule jobs are sequenced in desending order of magnitude of their respective processing times.

Sequence :

Job	A	E	C	B	D
Processing Time (Days)	7	6	5	4	2

Answer for Q.NO.51.

(a) Calculation of slack :

Number of days until due date is 1 week i.e. 5 days for all jobs

Job (1)	No. of days until/due date (2)	No. of day of work remaining (3)	Slack (Days) (4) = (2) – (3)
A	5	2	3
B	5	4	1

C	5	7	– 2
D	5	6	– 1
E	5	5	0
F	5	3	2

Sequence :

Job	C	D	E	B	F	A
Slack (Days) -	2	-1	0	1	2	3

Jobs are sequenced in ascending order of magnitude of respecive slack values.

(b) Calculation of Critical ratio :

$$= \frac{\text{Due Date . Date Now}}{\text{Lead Time Remaining}} = \frac{\text{DD . DN}}{\text{LTR}} = \frac{\text{Available time till due date}}{\text{Operation time still needed to complete the job}}$$

Critical ratio for job A = 5/2 = 2.5

Critical ratio for job B = 5/4 = 1.25

Critical ratio for Job C = 5/7 = 0.71

Critical ratio for job D = 5/6 = 0.83

Critical ratio for job E = 5/5 = 1.0

Critical ratio for job F = 5/3 = 1.67

Job having least critical ratio is given the first priority and so on.

Sequence :	C	D	E	B	F	A
Critical Ratio :	0.71	0.83	1.0	1.25	1.67	2.5

Answer for Q.NO.52.

(a) The least of all the times given in the table is for job 6 on machine B. So, perform job 6 in the end. It is last in the sequence. Now delete this job from the given data.

(b) Of all timings now, the minimum is for job 3 on machine A. So, do the job 3 first.

(c) After deleting job 3 also, the smallest time of 3 hours is for job 1 on machine B. Thus, perform job 1 in the end (before job 6).

(d) Having assigned job 1, we observe that the smallest value of 4 hours is shared by job 2 on machine A and job 5 on machine B. So, perform job 2 first and job 5 in the end.

(e) Now, the only job remaining is job 4, it shall be assigned the only place left in the sequence. The resultant sequence of jobs is, therefore, as follows:

3	2	4	5	1	6

This sequence is the optimal one. The total elapsed time, T, is obtained in Table 2.8.16 as equal to 36 hours

Table: Calculation of Total Elapsed Time (T)

Job	**Machine A**		**Machine B**	
	In	**Out**	**In**	**Out**

3	0	2	2	8
2	2	6	8	16
4	6	11	16	22
5	11	20	22	26
1	20	27	27	30
6	27	35	35	36

As shown in this table, the first job, job 3, starts at lime 0 on the machine A and is over by time 2, when it passes to machine B to be worked on till time 8. The job 2 starts on the machine A at time 2 as the machine is free at that lime. It is completed at time 6 and has to wait for 2 hours before it is processed on machine B, starting at time 8 when this machine is free, Similarly, the various jobs are assigned to the two machines and the in and out times are obtained.

5. PRODUCTIVITY MANAGEMENT AND QUALITY MANAGEMENT

Answer for Q.NO.1.

Monthly productivity per worker = 200 / 10 = 20 units

Answer for Q.NO.2.

Productivity = Actual production / Standard production

Standard production of hose complings per shift = 8 x 60 / 15 = = 32 pcs.

Productivity of industry A = 30 / 32= 15 / 16 and productivity of industry B = 20 / 32 = 5 / 8

If the productivity is expressed in percentage, the same for A is 15 / 16 = 100 = 93.75%

and productivity of industry B is 5 / 8 x 100 = 62.5%

Production per week of industry A = 30 × 6 = 180 nos. (Assuming the industry to work for one shift per day)

Production per week of industry B = 20 × 6= 120 nos. (Assuming the industry to work for one shift per day)

Answer for Q.NO.3.

(a) Hours worked per day = 8

Working days per month = 25

Hours worked per month = 25 × 8 = 200 hrs.

Machine time = 22 minutes

Operator time = 8 minutes

Total time per unit = 30 minutes = ½ hr.

No. of units that can be produced/month/operator = 200 / 1/ 2 = 400

As the no. of operator is 1, possible monthly production = 400 units. As the plant operates at 75% efficiency.

Monthly production = 400 × 75 / 100 = 300 units.

(b) If machine productivity is increased by 10% i.e. Machine time = 22 × 100

(100 +10) = 20 minutes.

Then, total time = 20 + 8 = 28 minutes

Monthly production = $\frac{400 \text{x} 30}{28} \text{x} \frac{75}{100} = 321 \text{units}$

(c) If operator efficiency reduced by 20% i.e.

Operator time = 8 × $\frac{(100+20)}{100}$ = 8 × 1.2 = 9.6 minutes.

Total time = 22 + 9.6 = 31.6 minutes.

Monthly production = $\frac{400 \text{x} 30}{31.6} \text{x} \frac{75}{100}$ = 284 units.

(Efficiency reduced by 20%. Instead of 100%, now 80% job is completed in 8 minutes. That means, operators time is increased to 10 minutes)

Answer for Q.NO.4.

No. of days per month = 25

Daily working hrs. = 8

No. of operators = 15

No. of Man days per month = 15 × 25 = 375 Man days.

Total working hrs. per month = 375 × 8 = 3,000

Hours lost in absentism in a month = 30 × 8 = 240

(a) Percent absentism = $\frac{240\text{hrs.x}100}{3000\text{hrs}} = 8\%$

(b) Efficiency of utilisation of labour = $\frac{\text{Standard labour hour to produce 240 units}}{\text{Total labour hour}} \text{x}100$

$= \frac{240\text{x}8}{3000}\text{x}100 = 64\%$

(c) Standard time required to produce 240units = 240 × 8 = 1920 labour-hours.

In November, man hours lost = 30 × 8 = 240

idle time (in hours) = 276

Total loss of time = 516 hours.

Productive hours available in November = 3000

Less, Total loss of time = (516)

Actual labour-hours = 2484 hours

$= \frac{\text{Std. Labour hrs}}{\text{Actual Labouthrs.}} = \frac{1920\text{x}100}{2484} = 77.3\%$

(d) 15 men produces 300 units,

Std. labour productivity = 300/15 = 20 units.

In November, overall productivity = 240/15 = 16 units. (Ans.)

i.e. productivity falls by 25%.

Answer for Q.NO.5.

Total standard minutes worked during the day = 30 × 15 = 450, working time = 8 – 2 = 6 hours = 360 minutes.

Performance = (450 × 100) /360 = 125% i.e incentive is payable on 25% which is above 100%

(i) Incentive bonus = 0.25 × 6 × 4 = Rs. 6 for six hours on measured work

(ii) Guaranteed wage for 8 hours = 8 × 4 = Rs. 32; Total earnings for the days

= Rs. (6 + 32) = Rs. 38

(iii) Net labour productivity = Output in units / Net man hours = 15 / 6 = 2.5 sets per hour.

Answer for Q.NO.6.

Month	No. of machines employed	Working hours	Machine hours	Production Units
January	400	220	88,000	99,000
February	550	180	99,000	1,00,000
March	580	220	1,27,600	1,25,000

P = Productivity per machine hour

= Number of units produced /Machine hours

For January P = 99,000/88,000 = 1.125

February P = 100,000 / 99,000 = 1.010

March P = 125,000 /127,600 = 0.980

Interpretation: Though the total production in number of units is increasing, the productivity is declining.

Answer for Q.NO.7.

Normal time per unit = Observed time / unit × Rating factor = 5 × (120/100) = 6minutes

Allowances = 30% of normal time = (30 × 6)/100 = 1.8 minutes

Standard time/unit = Normal time/unit + Allowances = 6 + 1.8 = 7.8 minutes / unit

Standard production in shift of 8 hours = (8 × 60)/7.8 = 61.54 units.

Answer for Q.NO.8.

Average time for Activity Element A = 0 15 + 0.25 + 0 17 / 3 = 0.19 min.

Average time for Activity Element B = 1 56 + 1.80 + 1 75 / 3 = 1.703 min.

Average time for Activity Element C = 0 20 + 0.10 + 0 15 / 3 = 0.15 min.

Computation of Normal Time

Activity Element	Average time (Mins)	Performance Rating (%)	Normal Time (Mins)	
(1)	(2)	(3)	(4) = (2) × (3) ÷ 100	So Normal Time for the job of packaging = 2.101 Mins
A	0.19	90	0.171	
B	1.703	105	1.788	
C	0.15	95	0.142	
Total	—	—	2.101	

$$\frac{\text{Normal Time}}{1-(\text{Allowance Factor}/100)} = \frac{2.101}{1-\frac{10}{100}} = 2.334\text{Mins}$$

Standard Production in a shift of 8 hours = $\frac{8 \times 60}{2.334}$ = 205.66 cartoons.

Answer for Q.NO.9.

(a) The processing time needed in hours to produce products A, B and C in the quantities demanded using the standard time data:

Product	Annual Demand (units)	Processing time (standard time in hours)	Processing time needed to produce demand quantity (hrs.)
A	325	5.0	325 × 5 = 1,625
B	450	4.0	450 × 4 = 1,800
C	550	6.0	550 × 6 = 3,300
			Total = 6,725 hrs.

(b) Annual production capacity of one machine in standard hours = 8 × 288 = 2,304 hours per year.

(c) Number of machines required = Work load per year / Production capacity per Machine = 6,725 / 2,304 = 2.90 machines = 3 machines.

Answer for Q.NO.10.

Percentage of working time = $\frac{2500-400}{2500} x100 = 84\%$

Actual working time in a study of 100 hours = 84 hours = 84 × 60 = 5040 mins.

Production — 6000 articles

Time required to produce an article = 5040 / 6000 = 0.84 mins

Of this Manual time = 0 84 x 2/ 3 ($\therefore$ Ratio of Manual to Machine activity elements = 2:1)

= 0.56 mins

Machine time = 0.84 × 1 / 3 = 0.28 min.

Normal Time of man = Time of man as per study × Rating Factor / 100

= 0 56 x 115 / 100 = 0.644 min.

Normal Time of machine = 0.28 min.

Allowances for man = 12% of Normal time of Man = 0.12 × 0.644 = 0.077 min

Standard Time for Man to produce an article = Normal Time of Man + Allowances

= 0.644 + 0.077 = 0.721 min.

Standard Time for machine = 0.28 min.

Standard Time to produce an article = 0.28 + 0.721 = 1.001 mins.

Standard time required to produce 42000 articles = 42000 × 1.001 = 42042 mins. = 700.7 hours.

No. of days/month - 25, Daily working hours - 8, No. of workers - 5

Total available working hours/month = 5 × 25 × 8 = 1000

Actual working hours/month = 1000 × 0.93 [Since Absentism = 7%]

= 930

Efficiency of utilisation of Labour = Standard time to produce 42000 article / Total available hours x 100

= 700 7 / 1000 x 100 = 70.07 %

Productive efficiency of Labour =

Standard time to produce 42000 article / Actual working hours x 100

= 700 7 / 930 x 100 = 75 34%

Answer for Q.NO.11.

Productivity per worker = 2500/100 = 25 tonnes.

Answer for Q.NO.12.

Productivity =Actual production/Standard production

Standard production of tobacco plant is = 8 × 60 / 10 = 48 packets.

Productivity of plant located in state Y = 40/48 = 0.833 (83.33%)

Productivity of plant located in state Z = 55/48 = 1.146 (114.6%)

Now, expected productivity of plant in Y = 40 × 7 = 280 Packets (if it works for 7 days with one shift)

And, expected productivity of plant in Z = 55 × 7 = 385 Packets (if it works for 7 days with one shift)

Answer for Q.NO.13.

Working hours per month = 22 × 8 = 176 hrs.

No. of units that can be produced/month by the operator = 176 × 60/38= 277.89 approx 278.

a. Now if the plant efficiency = 65% and since there is only one operator and its efficiency is 100% then expected production of spare parts = 277.89 × 0.65 = 180.62 Units.

b. If the plant efficiency increases by 20% new output will be

New machine time is 28 × 100/120 = 23.33 minutes, and then the total time = 10 + 23.33 = 33.33 min

New monthly production = 277.89 × (38/33.33) × 0.65 = 205.93 Units

c. If the operator's efficiency is reduced by 30% then new production will be

New Operator's time = 10 × (130/100) = 13 min

So new total time = 13 + 28 = 41 minutes.

Now new monthly production = 277.89 × (38/41) × 0.65 = 167.41 units

Answer for Q.NO.14.

No of mandays per month = 20 × 24 = 480 Man days

Total working hr per month = 480 × 8 = 3840.

Hr lost in absenteeism in a month = 28 × 8 = 224

1. Efficiency of utilisation of manpower = (250 × 8 / 3840) × 100 = 52.08%

2. Absenteeism = (224 hr/3840hr) ×100 = 5.833%.

3. 20 men logs 290 calls in a month

So the St. manpower productivity = 290/20 = 14.5 calls.

In the current month overall productivity = 250/20 = 12.5 calls

So, the productivity has fallen from 14.5 to 12.5 i.e. 13.8%

Answer for Q.NO.15.

Quarter	No of Employees	Working Hours	Man Hours	Business Achieved (Rs.)	Productivity
Q1	1600	800	1280000	1000000	0.78125
Q2	1500	750	1125000	1024000	0.910222
Q3	1700	775	1317500	1300000	0.986717
Q4	2000	900	1800000	1200000	0.666667

Man hour of Q1 = No of Employee × Working Hours = 1600 × 800 = 1280000

Productivity in Q1 = 1000000/1280000 = 0.78125

From the above table we can say that the productivity of Q3 is best then follows Q2 then Q1 and the least is Q4.

Answer for Q.NO.16.

Normal time per unit = Observed time / unit × Rating factor = 7 × (110/100) = 7.7 minutes

Now, Allowances = 25% of normal time = (25 × 7.7)/100 = 1.925 minutes.

So, Standard time/unit = Normal time/unit + Allowances = 7.7 + 1.925 = 9.625 minutes / unit

Hence, Standard production in shift of 8 hours = (8×60)/9.625 = 49.89 units (50 Units approx.)

Answer for Q.NO.17.

(a) Product 1 → Processing time needed toproduce demand quantity (hrs.) = Annual Demand × Processing time (Standard time in hr) = 425 × 4 = 1700 hrs

Product 2 → Processing time needed to produce demand quantity (hrs.) = 429 × 5 = 2145 hrs

Product 3 → Processing time needed to produce demand quantity (hrs.) = 546 × 5.5 = 3003 hrs.

Hence total time needed= 6848hrs.

(b) Annual production capacity for a single machine = 8 × 200 =1600 hrs for a year.

(c) Minimum number of machines required = 6848/1600 = 4.29 (5 machine Approx.)

Answer for Q.NO.18.

Percentage of working time = ((3000-500)/3000) × 100 = 83.33%

Actual working time in a study of 150 hrs = 150 × 0.8333 × 60 = 7500 min.

Production = 7000 units.

Time required for one unite to produce = 7500/7000 = 1.0714 min

So, manual time on this is 1.0714 × (3/5) = 0.643 mins and machine time is 1.0714 × (2/5) = 0.43 mins.

Normal time of labour = time of labour as per study × Rating Factor/100 = 0.643 × 120/100= 0.772 min.

And normal time of machine = 0.43 min.

Now, if allowance is considered which is 11% of normal time which bring the standard time for the labour to produce = 0.772 × 1.11 = 0.857 min.

Hence Standard time required to produce a product = 0.857 + 0.43 = 1.286 min.

And. Standard time required to produce 49000 units = 49000 × 1.286 = 63,038 min = 1050 hrs.

Now, total available working hrs for 24 working day of 8 hrs shift of 7 labours = 1344 hr in a month

Taking absenteeism in consideration actual working hour left = 1344 × 0.94 = 1263.4 hrs.

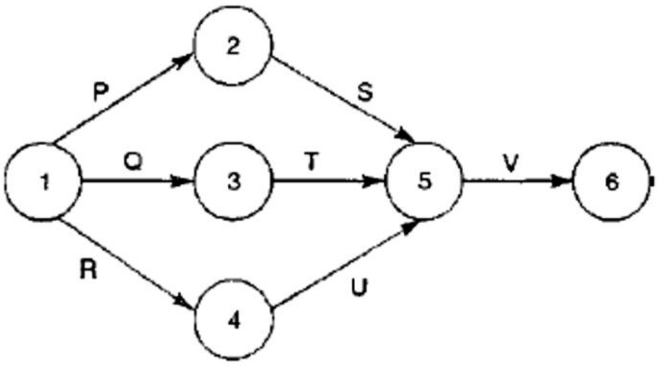
Efficiency of utilisation of Labour= 1050/1344 = 0.7813 (78.13%)

And

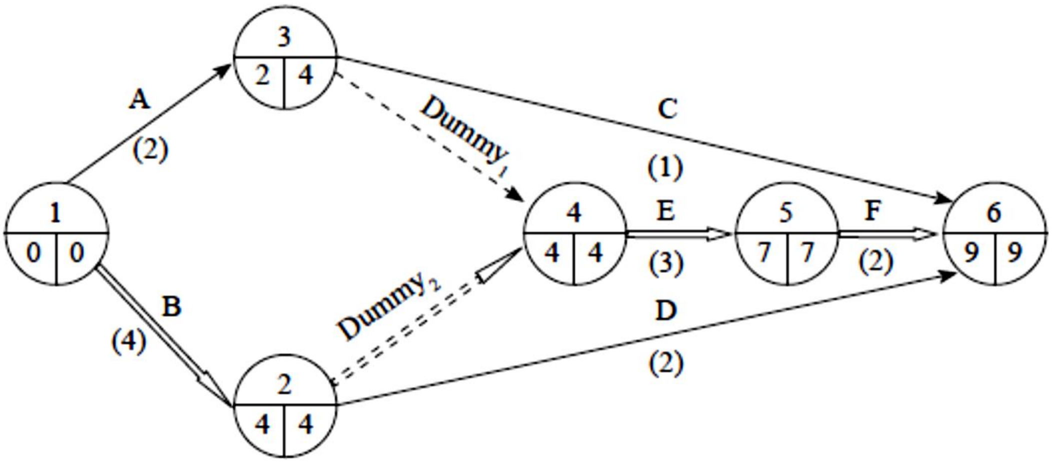
Productive efficiency of Labour = 1050/1263.4 = 0.8312 (83.12%)

6. PROJECT MANAGEMENT, MONITORING AND CONTROL

Answer for Q.NO.1.



Answer for Q.NO.2.



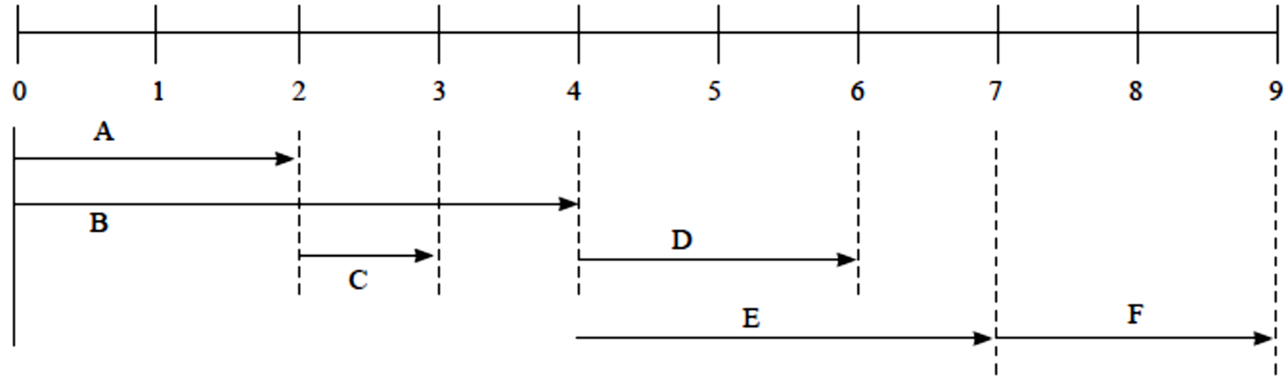
Critical Path B – Dummy2 – E – F

Minimum duration of the project = 9 days

Table: Activity Relationship

Activity	t	ES (EF- t)	EF	LS (LF- t)	LF	Event Slack (LS-ES) (LF-EF)	On Critical Path
A	2	0	2	2	4	2	No
B	4	0	4	0	4	0	Yes
C	1	4	5	8	9	4	No
D	2	4	6	7	9	3	No
E	3	4	7	4	7	0	Yes
F	2	7	9	7	9	0	Yes

(2) Gantt Chart for Early Start Schedule



(3) Peak requirement of money will occur during simultaneous occurance of Activities.

From the Network diagram above, it can be said that the following Activities need to occur simultaneously.

(i)	A & B	—	Either during the days 1 & 2 or during the days 3 & 4 of Project Duration, which will require (Rs. 50 for A + Rs. 50 for B) per day i.e. Rs. 100 per day
(ii)	B & C	—	Either on day 3 or on day 4 of the project and it will require (Rs. 50 for B + Rs. 40 for C) per day i.e. Rs. 90 per day
(iii)	C, D & E	—	During day no. 5 or day no. 6 and cost is Rs. (40 + 100 + 100) = Rs. 240 per day
(iv)	C, D & F	—	During day no. 8 or day no. 9 and cost is Rs. (40 + 100 + 60) = Rs. 200 per day
(v)	D & E	—	During day nos. 5 & 6 or 6 & 7. Cost is Rs. (100 + 100) = Rs. 200 per day
(vi)	D & F	—	During day nos. 8 & 9. Cost = Rs. (100 + 60) = Rs. 160 per day
(vii)	C & E	—	Either on day no. 5 or 6 or 7. Cost to be incurred = Rs. (40 + 100) = Rs. 140 per day

From above we can say that C can occur by using either of the options (ii), (iii), (iv) & (vii). As cost fo option (ii) is least one should decide fo it at a cost of Rs. 90 per day.

Similarly D can occur by either of the option (iii), (iv), (v) & (vi) above. As (vi) is the least cost option of all these, one should go for it at a cost of Rs. 160 per day.

Hence the Project Activities should follow the sequence given below :-

(a) A & B to start at their Earliest Time (i.e 0) and occur simultaneously till day 2 @ Rs. 100 per day

(b) C can start either at its Earliest Time (i.e. 2) or on day 3 and occur simultaneously with B either on day 3 or 4 @ Rs. 90 per day

(c) E being Critical Activities must have to start at it's earliest time (i.e. 4) and occur @ Rs. 100 per day

(d) F being Critical Activity has to start on Earliest Time (i.e. 7) and will occur concurrently with D is the during days 8 & 9 @ Rs. 160 per day.

Hence peak requirement of money is Rs. 160 per day and it will occur at days 8 and 9.

Answer for Q.NO.3.

Steps:

1. Moving forward, find EF times (choosing the Maximum at activity intersection)
2. Maximum EF = LF = Critical Path Time
3. Return path find LF (Choosing the Minimum at activity intersection)
4. Note LF, EF from network (except activity intersections)

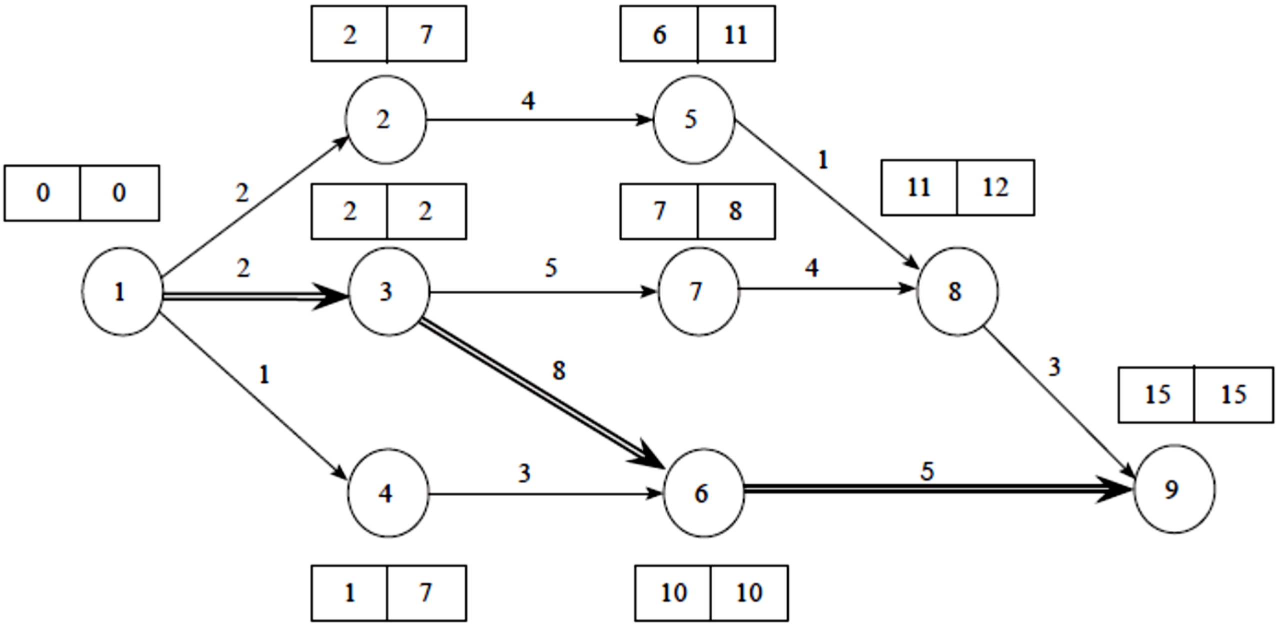


Table: Activity Relationship

Activity	Duration Months (tij)	Earliest Start (ESij)	Earliest Finish (EFij = ESij + tij)	Latest Start (LSij = LFij – tij)	Latest Finish (LFij)	Total Float (TFij = LSij + ESij = LEij – EFij)
1 -2	2	0	2	5	7	5
1 -3	2	0	2	0	2	0
1 -4	1	0	1	6	7	6
2-5	4	2	6	7	11	5
3-6	8	2	10	2	10	0
3-7	5	2	7	3	8	1
4-6	3	1	4	7	10	6
5-8	1	6	7	11	12	5
6-9	5	10	15	10	15	0
7-8	4	7	11	8	12	1
8-9	3	11	14	12	15	1

Critical path is 1-3-6-9 with duration 15 months

Minimum number of cranes

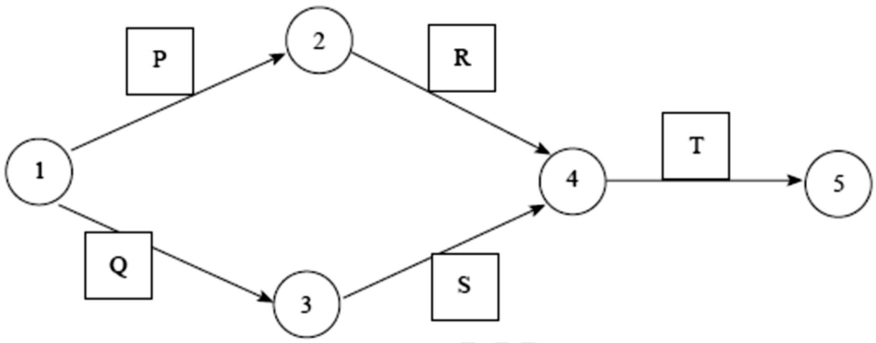
- Finish 3 — 7 at its earliest finish time 7 with one crane

- Finish 2 — 5 at its latest finish time 7 + 4 =11 with the same crane by starting the activity at its latest start time 7
- Finish 5 — 8 at its latest finish time 11 + 1 = 12 with the same crane by starting the activity at its latest start time 11
- Finish 8 — 9 at its latest finish time 12+ 3=15 with the same crane by starting the activity at its latest start time 12

Therefore, one crane will be sufficient if start time of the following activities are:

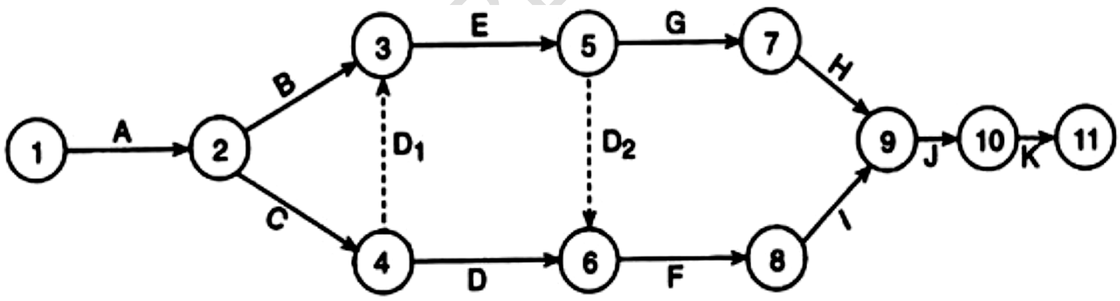
- Activities 2-5 — 7
- Activities 5-8 — 11
- Activities 8-9 — 12

Answer for Q.NO.4.



Answer for Q.NO.5.

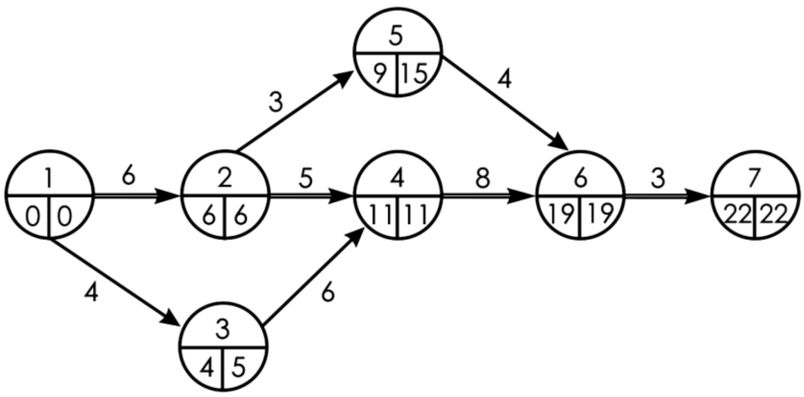
The network diagram will be as follows:



Where D1 and D2 are dummy activities

Answer for Q.NO.6.

(i) The network for normal activity times indicates a project time of 22 days with the critical path 1-2-4-6-7.



(ii) Normal project duration is 22 days and the associated cost is as follows:

Total cost = Direct normal cost + Indirect cost for 22 days.

= 4,700 + 100 × 22 = Rs. 6,900.

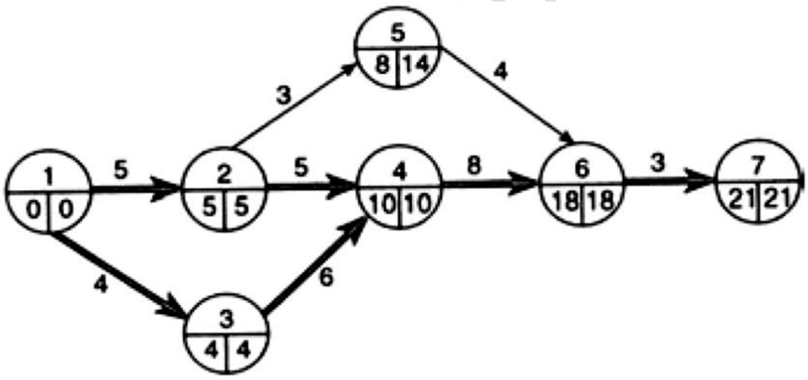
(iii) For critical activities, cost - slope is given below:

Critical activity	**Cost-slope* (Rs./day)**	
1-2	$\frac{1000-600}{6-4}=200$	$\frac{\text{Crash Cost Normal Cost}}{\text{Normal Time Crash Time}}$
2-4	$\frac{1500-500}{5-3}=500$	
4-6	$\frac{3000-800}{8-4}=500$	
6-7	$\frac{800-450}{3-2}=350$	

*Cost slope Crash Cost Normal Cost Normal Time Crash Time

Of the activities lying on the critical path, activity 1—2 has lowest cost slope Therefore, we shall first crash this activity by just one day.

Duration = 21 days, and cost = 4700 + 1 × 200 + 100 × 21 = Rs. 7000.



Other activities too have become critical. Now we have 2 critical paths:

1→2→4→6→7 and 1→3→4→6→7.

To reduce duration of the activity further, we shall have to reduce duration of both the paths. We have following

alternatives:

Crash activity 6 — 7 by 1 day at a cost of Rs. 350.

Crash activity 4 — 6 by 4 days at the cost of Rs. 550 per day.

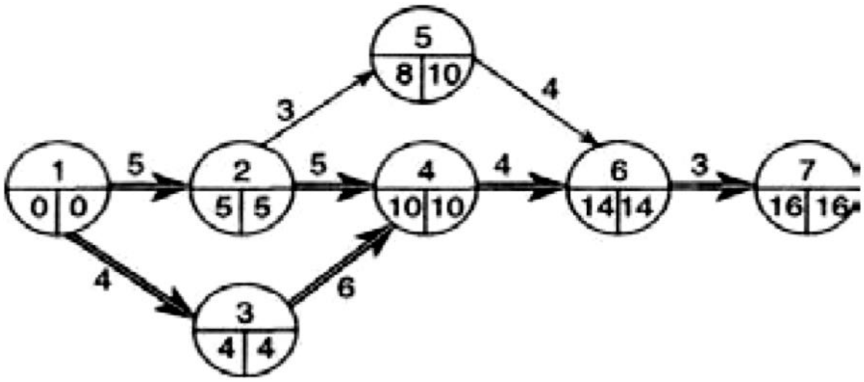
Crash activities 1—2 and 1 — 3 by 1 day each at a cost of Rs. (200 + 700) = Rs. 900.

Crash activities 2 — 4 and 3 — 4 by 2 days each at a cost of Rs. (500 + 550) = Rs. 1050/day.

Thus, we shall first crash activities 6 — 7 by 1 day and then activity 4 — 6 by 4 days.

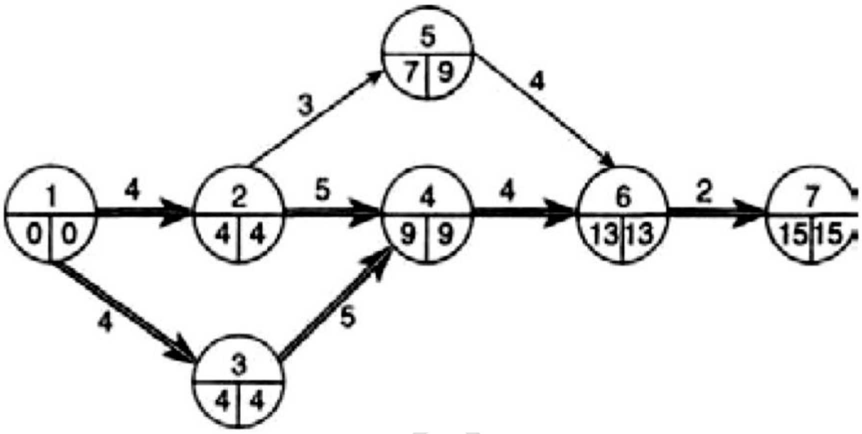
On crashing activity 6 — 7 by 1 day, cost = 4900 + 350 × 1 + 100 × 20 = Rs. 7250, and duration = 20 days. Next we crash 4—6 by 4 days.

Cost = 5250 + 550 × 4 + 100 × 16 = Rs. 9050. Duration = 16 days.



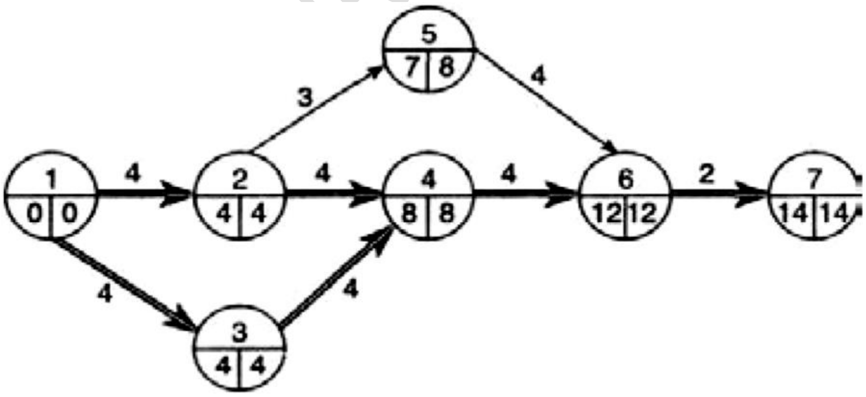
Next we crash activities 1—2 and 3—4 by 1 day each.

Cost = 7450 + 200 × 1 + 550 × 1 + 100 × 15 = Rs. 9700.



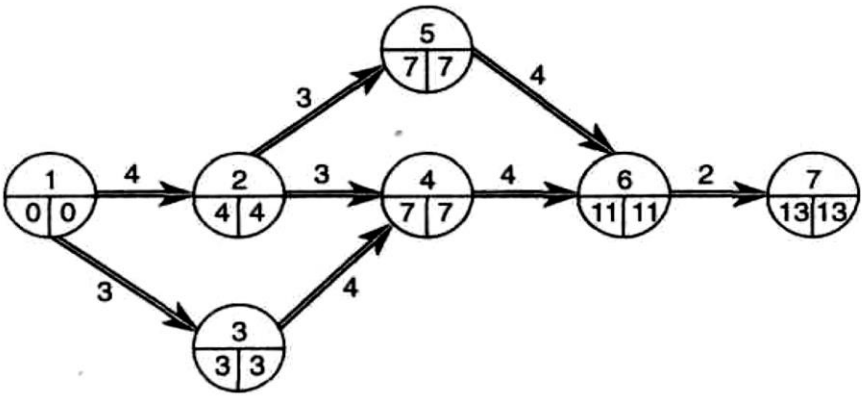
Next we crash activities 2 4 and 3 4 by 1 day each.

Cost = 8200 + 500 × 1 + 550 × 1 + 100 × 14 = Rs. 10,650. Duration = 14 days.



We crash activities 1—3 and 2—4 by 1 day each.

Cost = 9250 + 700 × 1 + 500 × 1 + 100 × 13 = Rs. 11,750 Duration = 13 days.

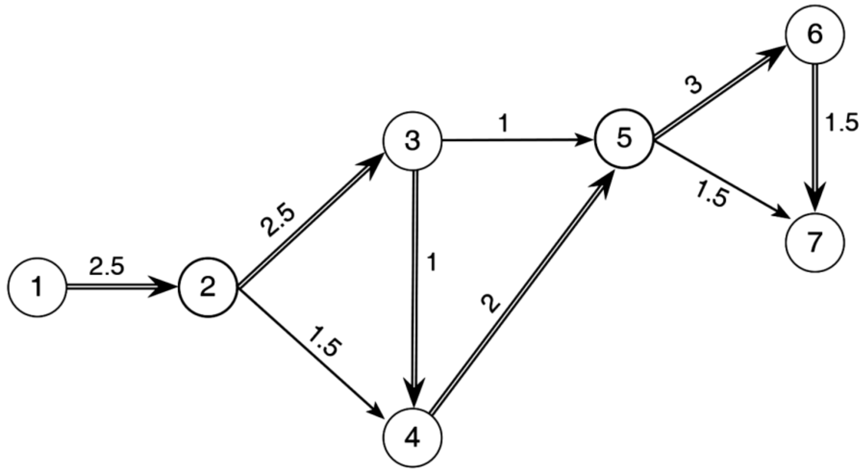


Now there are three critical paths:

1—2—5—6—7, 1—2—4—6—7, 1—3—4—6—7

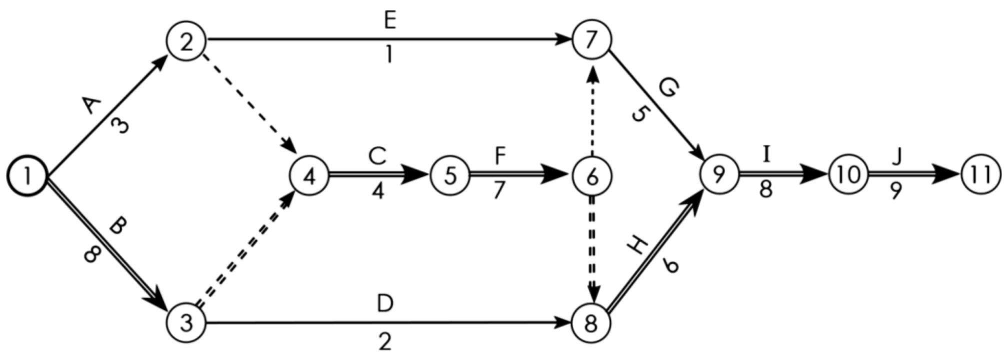
Also, no further crashing is possible. Hence minimum duration of the project =13 days with cost Rs. 11,750

Answer for Q.NO.7.

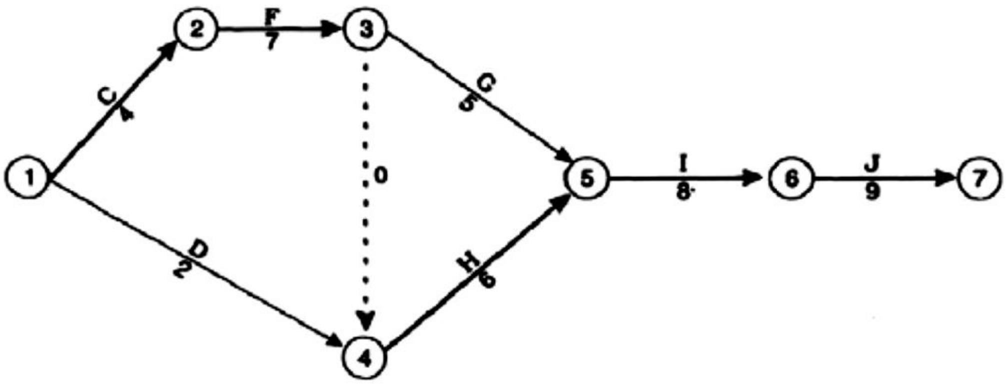


Paths	Duration
1-2-3-5-6-7	2.5+2.5+1+3+1.5 = 10.5
1-2-3-5-7	2.5+2.5+1+1.5 = 7.50
1-2-3-4-5-6-7	2.5+2.5+1+2+3+1.5 = 12.5 (Critical path)
1-2-3-4-5-7	2.5+2.5+1+2+1.5 = 9.5
1-2-4-5-7	2.5+1.5+2+1.5 = 7.5
1-2-4-5-6-7	2.5+1.5+2+3+1.5 = 10.5

Answer for Q.NO.8.



Paths	Duration (weeks)	Paths	Duration (weeks)
1-2-7-9-10-11	26	1-3-4-5-6-7-9-10-11	41
1-2-4-5-6-7-9-10-11	36	1-3-4-5-6-8-9-10-11	42
1-2-4-5-6-8-9-10-11	37	1-3-8-9-10-11	33
Critical Path: BCFHIJ. Duration 42 weeks.			



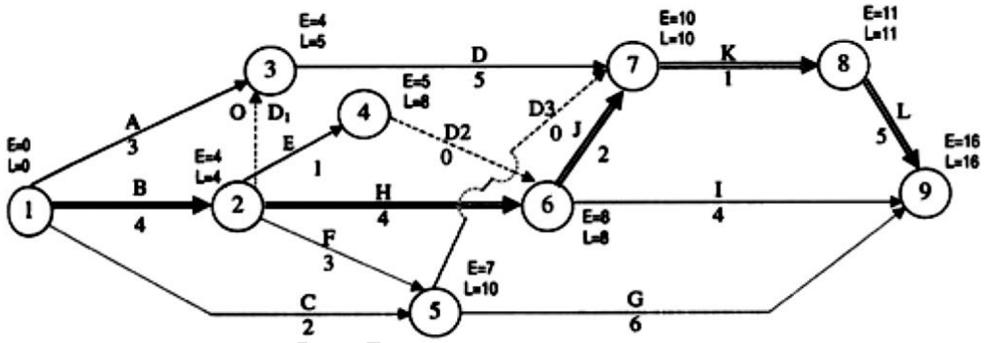
Paths 1-2-3-5-6-7 1-2-3-4-5-6-7 1-4-5-6-7

Duration (weeks) 33 34 **Critical Path: CFHIJ** 25

For completing the project as per original schedule, the project activities on the critical path should be reduced by 2 weeks. For example, we may reduce any one of the activities CFHIJ by 2 weeks or any two activities by one week each.

Answer for Q.NO.9.

Network Diagram:



Network Table:

Activity	Duration	EST	LST	EFT	LFT	Total Float	Free Float	Independent Float
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	3	0	2	3	5	2	2 – 1= 1	1 – 0 = 1
B	4	0	0	4	4	**0**	0	0
C	2	0	8	2	10	8	8 – 3 = 5	5 – 0= 5
D1	0	4	5	4	5	1	1 – 1 = 0	0
D	5	4	5	9	10	1	1 – 0 = 1	1 – 1 = 0
E	1	4	7	5	8	3	3 – 3 = 0	0
F	3	4	7	7	10	3	3 – 3 = 0	0
G	6	7	10	13	16	3	3 – 0 = 3	3 – 3 = 0
D2	0	5	8	5	8	3	3 – 0 = 3	3 – 3 = 0
H	4	4	4	8	8	**0**	0	0
I	4	8	12	12	16	4	4 – 0= 4	4 – 0= 4
J	2	8	8	10	10	**0**	0	0

D3	0	7	10	7	10	3	3 – 0 = 3	3 – 3 = 0
K	1	10	10	11	11	**0**	0	0
L	5	11	16	16	16	**0**	0	0

Critical path is B – H – J – K – L. Expected Duration = 16 days

The columns are updated in the following order as under:

1. Activity (including Dummies) are listed from the Question and network Diagram

2. Duration (including Dummies) are listed from the Question and Network Diagram

3. EST = E value of LHS/ Tail Event from Diagram.

6. LFT = L value of RHS/ Head Event from Diagram.

5. EFT = EST + Duration as per Column (2). Hence Column (5) = Column (3) + Column (2)

4. LST = LFT - Duration as per Column (2). Hence column (4) = Column (6) – Column (2)

7. Total Float = [LET – EFT] or [LST – EST] = [Col.(6) – Col.(5)] or [Col.(4) – Col.(3)]

8. Free Float = Total Float – Head Event Slack i.e. [Col.(7) – difference between L and E of RHS Event].

Note: If Total Float is Zero, Free Float is also equal to Zero If a negative value is derived, it is restricted to zero.

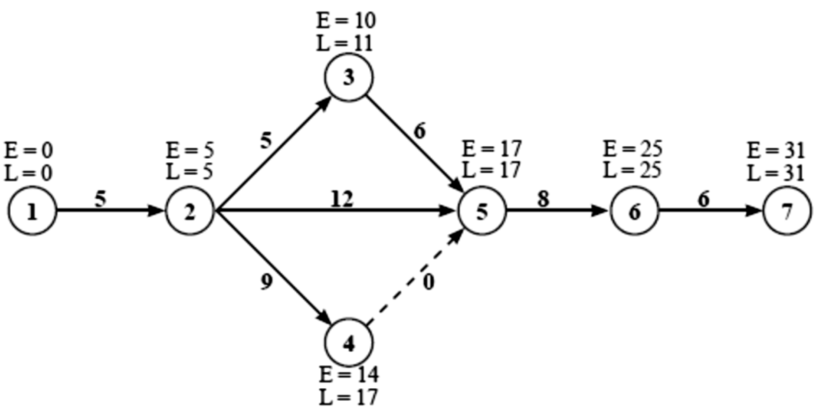
9. Independent Float = Free Float – Tail Event Slack i.e. [Col (8) – Difference between L and E of LHS Event].

Note: If Free Float is Zero, Independent Float is also equal to Zero. If a negative value is derived, it is restricted to zero.

Note:

- The activities whose Total Float is Zero are Critical Activities. These Total Floats are circled and the respective activities are indicated by double in the network diagram.
- Dummy Activities may or may not lie on the critical path. However, in this question, the dummy activities do not fall on the Critical Path.

Answer for Q.NO.10.



Paths →	1-2-5-6-7 (Let's denote this by A)	1-2-3-5-6-7 (Let's denote this by B)	1-2-4-5-6-7 (Let's denote this by C)
Duration	31 hours	30 hours	28 hours
The critical path is A. Its duration is 31 hours			

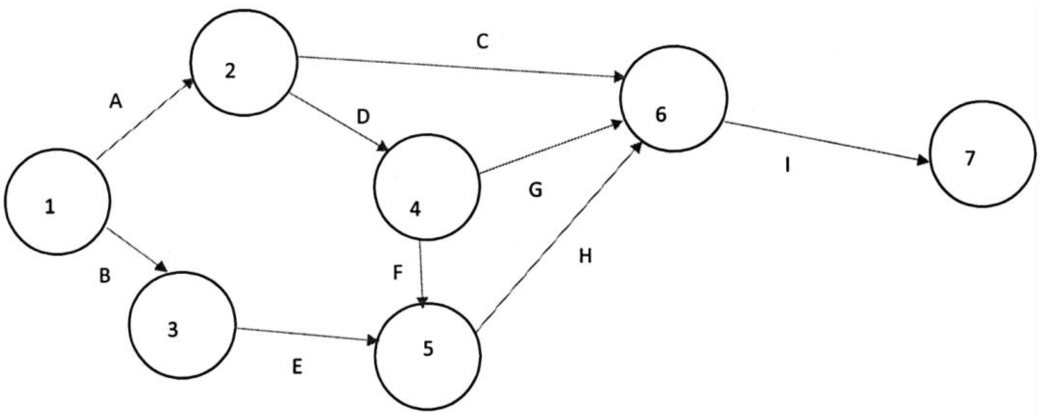
Answer for Q.NO.11.

CPM originated from construction project while PERT evolved from R & D projects. Both CPM and PERT share the same approach for constructing the project network and for determining the critical path of the network.

There is some basic differences between PERT and CPM

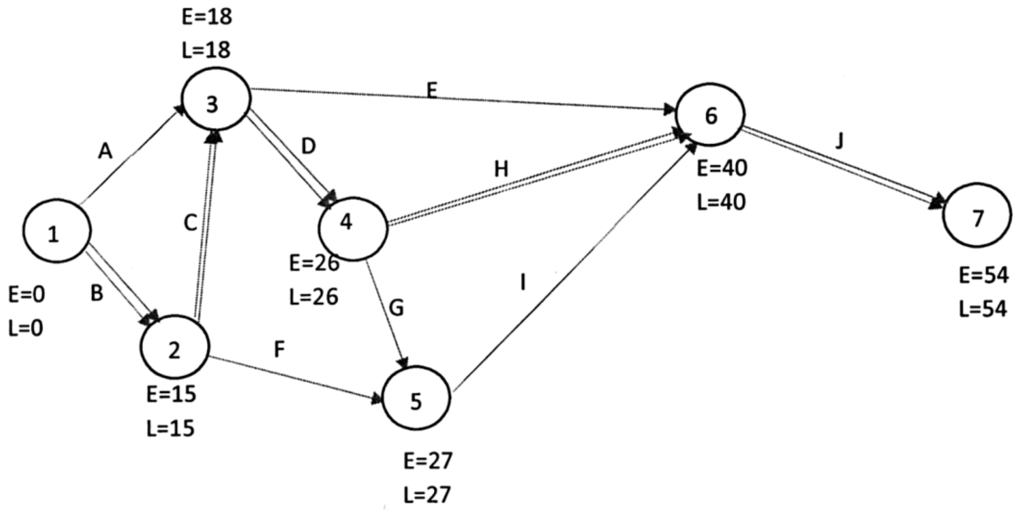
PERT	CPM
1. Time estimate is probabilistic with uncertainty in time duration. Three time estimates.	1. Time estimate is deterministic with known time durations. Single time estimate
2. Event oriented	2. Activity oriented
3. Focused on time	3. Focused on time-cost trade off
4. More suitable for new projects	4. More suited for repetitive projects

Answer for Q.NO.12.



Answer for Q.NO.13.

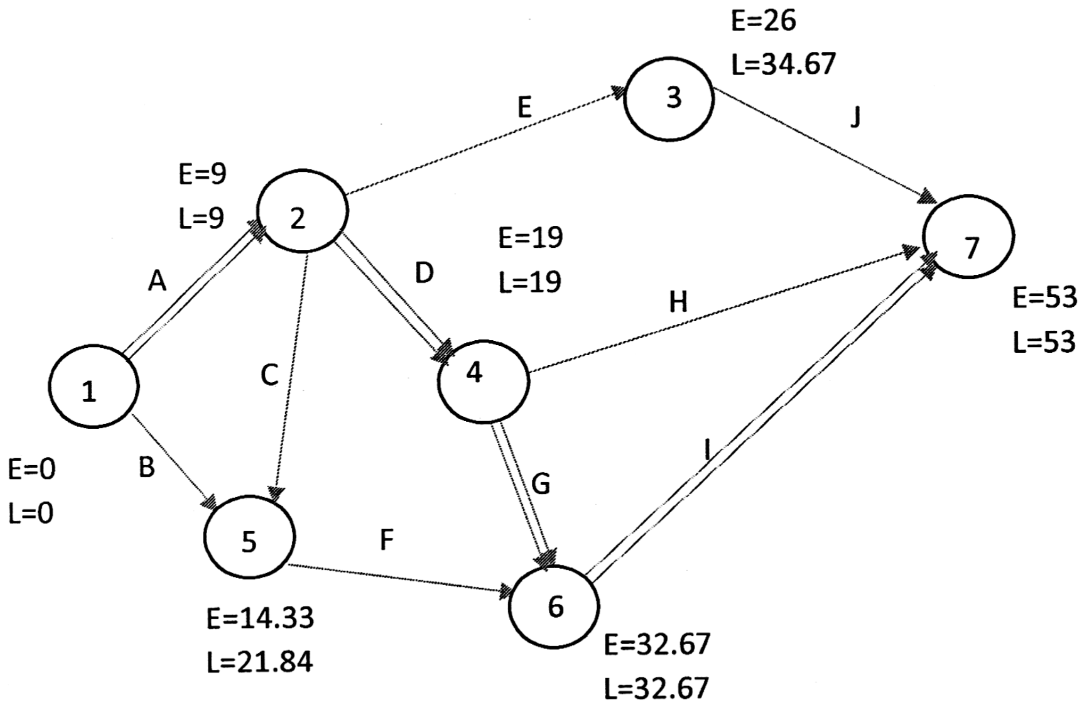
Activity (i-j)	Time (tij)	Earliest Start (ESTij)	Earliest Finish (EFTij = ESTij + tij)	Latest Start (LSTij = LFTij- tij)	Latest Finish (LFTij)	Total Float (TFij = LSTij + ESTij = LETij - EFTij)
A(1-3)	15	0	3	15	18	3
B (1-2)	15	0	0	15	15	0
C(2-3)	3	15	15	18	18	0
D (3-4)	8	18	18	26	26	0
E (3-6)	12	18	28	30	40	10
F(2-5)	5	15	32	20	37	17
G (4-5)	1	26	36	27	37	10
H (4-6)	14	26	26	40	40	0
1 (5-6)	3	27	37	30	40	10
J (6-7)	14	40	40	54	54	0



Critical Path 1-2-3-4-6-7

Critical Activity: B -C -D -H-J.

Answer for Q.NO.14.



Activity	Optimistic time (to)	Most likely Time (tm)	Pessimistic time (tp)	a2= (tp-to/6)^2	te =to+4tm+tp/6
1-**2**	6	9	12	1.00	9.0
1-5	4	7	8	0.44	6.7
2-3	14	17	20	1	17.0
2-4	7	10	13	1	10.0
2-5	3	5	9	1	5.33
3-7	13	18	25	4	18.33
4-6	10	14	16	1	13.67
4-7	12	15	18	1	15.00

5-6	9	11	12	0.25	10.83
6-7	17	20	25	1.78	20.33

The critical path is 1 - 2 - 4 - 6 - 7

Variance of the critical path = 1 + 1 + 1.78 = 3.78

SD of the critical path = SD of the network diagram = _3.78_ _ 1.944

Answer for Q.NO.15.

te = Expected time

A = Optimistic time;

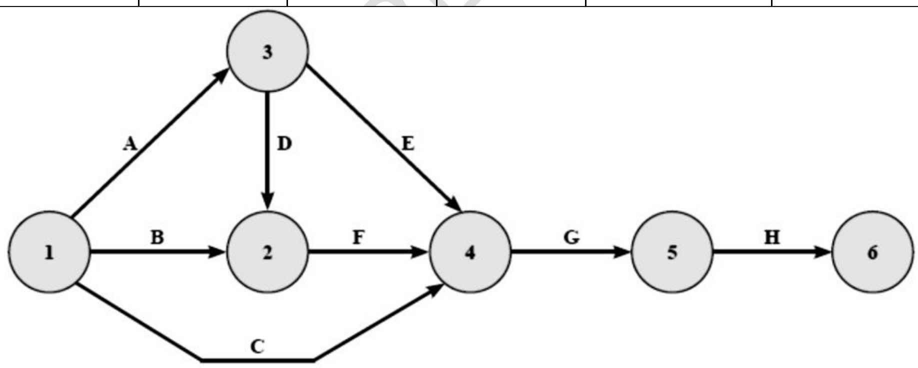
M = Most likely time;

B = Pessimistic time

Activity	A	M	B	Te	Variance
A	2	3	4	3	1/9
B	6	10	20	11	49/9
C	2	4	6	4	4/9
D	2	3	10	4	16/9
E	1	1	1	1	0
F	4	5	6	5	1/9
G	5	12	25	13	100/9
H	6	10	20	11	49/9

$t_e = (A + 4M + B)/6$

$\text{Variance}(t) = [(B - A)/6]^2$



Activity	A	M	B	Te	Variance	EST	EFT	LST	LFT	TF
A	2	3	4	3	1/9	0	3	12	15	12
B	6	10	20	11	49/9	0	11	0	11	0
C	2	4	6	4	4/9	0	4	12	16	12
D	2	3	10	4	16/9	11	15	11	15	0
E	1	1	1	1	0	15	16	15	16	0
F	4	5	6	5	1/9	11	16	11	16	0
G	5	12	25	13	100/9	16	29	16	29	0
H	6	10	20	11	49/9	29	40	29	40	0

There are two Critical Path

(i) 1---2---3---4---5---6 = (11 + 4 + 1 + 13 + 11) = 40

(ii) 1---2---4---5---6 = (11 + 5 + 13 + 11) = 40

As both the critical path suggest for both cases 40 days required to complete the project, so we calculate the

standard deviation of critical path

CSD1 = $\sqrt{\frac{49}{9}+\frac{16}{9}+0+\frac{100}{9}+\frac{49}{9}}$ = 14.9/3 = 4.966 (Approx.)

CSD2 = $\sqrt{\frac{49}{9}+\frac{1}{9}+\frac{100}{9}+\frac{49}{9}}$ = 14.1/3 = 4.7 (Approx.)

Here, CSD_2 performing better than CSD1, so we select the 2nd Critical Path.

Then here, $\mu = 40$, $\sigma = 4.7$

Since in 44 day taken then the percentage of work done

$P(T = 44) = P((T-\mu)/\sigma <= ((44-40)/4.7))$

$P(Z <= 4/4.7)$

$P(Z <= 0.8) = 0.78814$

Nearly 79% of the project will be completed during 44 days.

For the completion of 100% of the project we can take the 3 sigma limit

$P(T <= n)$

$P((T-\mu)/\sigma <= (n-\mu)/\sigma) = P(Z <= 3)$

$P(Z <= (n-40)/4.7) = P(Z <= 3)$

$n = 4.7 \times 3 + 40$

n = 54 day (Approx)

7. ECONOMICS OF MAINTENANCE AND SPARES MANAGEMENT

Answer for Q.NO.1.

Expected time before failure.

= 0.20×1 + 0.15×2 + 0.15×3 + 0.15×4 + 0.15×5 + 0.20×6 = 3.5 months

Therefore number or repair/machine/annum = 12/3.5

Considering 20 machines and Rs. 150 to attend a failed machine the yearly cost of servicing

= 12/3.5 × 20 × 150 = Rs. 10286.

Answer for Q.NO.2.

Converting the frequencies to a probability distribution and determining the expected cost/month of breakdowns we get:

No. of breakdowns (x)	Frequency in months (f)	Probability (p = f/Σf)	Expected no. of breakdowns (px)
0	2	0.083	0.000
1	8	0.333	0.333
2	10	0.417	0.834
3	3	0.125	0.375
4	1	0.042	0.168
	Σf = 24	Σp = 1	Total 1.710 = Σpx

Expected Breakdown cost per month; Expected no. of breakdowns per month × cost of each breakdown = 1.710 × Rs. 2800 = Rs. 4788.

Preventive maintenance cost per month: -

Average cost of one breakdown/month = Rs. 2, 800

Maintenance contract cost/month = Rs. 1,500

Total = Rs. 4,300

Thus, preventive maintenance policy is suitable for the firm.

Answer for Q.NO.3.

The total test time = (100 tubes) × 2000 hours = 200,000 tube-hours.

There are two tubes which have failed and hence the total time is to be adjusted for the number of hours lost due to the failures during the testing.

The lost hours are computed as = 2 × 2000 / 2 = 2000 hours.

The assumption is made here is that each of the failed tubes have lasted an average of half of the test period.

Therefore, the test shows that there are two failures during (2,00,000 – 2000) = 1,98,000 tube hours of testing.

During 365 days a year (four hours a day) for 10,000 tubes the number of expected failures

1,98,000 / 2 ×10,000 ×365 × 4 = 147.47 =148 tubes approximately.

Mean time between failures = 1,98,000 tubes hrs. of testing / 2 failure

= 99,000 tubes hours per failure = 99,000 / 4 × 365 = 67.8 tubes year per failure

Answer for Q.NO.4.

The mean time before failure is 5.4 months and the expected cost with no preventive maintenance would be 100 × 50 / 5.4 = Rs. 925.93 per month. The following calculations show B_j, the expected number of breakdowns between preventive maintenance intervals, for the possible intervals, that may be considered.

B1 = MP1 = 50 (0.10) = 5

B2 = m (P1 + P2) + B1P1 = 50(0.10+0.05) + 5(0.10) = 8

B3 = 50 (0.10 + 0.05 + 0.05) + 8 (0.10) + 5 (0.05) = 11.05

Accordingly, B4 = 16.75, B5 = 25.63, B6 = 35.5, B7 = 48.72, B8 = 63.46.

The costs of various preventive maintenance intervals are summarised in the table below :

Cost of alternative preventive maintenance intervals –

Number of months between preventive services (j)	Bj Expected Number of Breakdown in j months	Expected cost/ month to Repair Breakdown CR×Bj/j	Cost per month for preventive service every j month CR(M)/j	Total expected cost per month of preventive maintenance and repair
(1)	**(2)**	**(3)**	**(4)**	**(5)**
1	5.00	500.00	1000.00	1500.00
2	8.00	400.00	500.00	900.00
3	11.05	368.33	333.33	701.66
4	16.75	418.75	250.00	668.75
5	25.63	512.60	200.00	712.60
6	35.50	591.67	166.67	758.34
7	48.72	696.00	142.86	838.86
8	63.46	793.25	125.00	918.25

A policy of performing preventive maintenance every 4 months results in the lowest average cost, about Rs. 669.

This amount is Rs. 257 per month less than the Rs. 926 expected cost without preventive maintenance. This policy would reduce the costs by (257 ÷ 926) × 100 = 27.75% below the cost of repairing the machines only when they breakdown.

Answer for Q.NO.5.

Buffer stock is required to cover the lead time only, i.e. to cover one month's period.

Mean consumption rate = 5 per month

Referring to the Poisson distribution table for a = 5, we have for x = 7. ... Cumulative probability = 0.867

x = 8 ... Cumulative probability = 0.932

Thus, with seven items only 86.7 per cent service level is attained; with eight items 93.2 per cent service level is obtained. Since one would err on the higher side of the service level, the value of x = 8 is chosen.

This means, the amount of spares stock that has to be kept must correspond to a maximum demand rate Dmax of eight during the lead time. In other words we should keep a Buffer Stock = Dmax – Daverage during a lead time = 8 - 5 = 3 items.

Thus, buffer stock desired is three numbers of the given spare part.

Answer for Q.NO.6.

The expected cost of down-time

= (Probability of failure) × (Cost when break-down occurs)

= (1 – 0.990) × (Rs. 2 crore) = Rs. 2 lakh

However, the cost of procuring the spare now is Rs. 10 lakh. Therefore, expected cost of downtime is less than the cost of spare; hence the spare need not be bought along with the equipment.

Answer for Q.NO.7.

Step 1 : To determine the expected number of breakdowns per year:

No. of breakdowns (x)	Frequency of breakdowns in days i.e, f(x)	Probability distribution of breakdowns P(x)	Expected value of breakdowns X P(x)
0	40	40/300 = 0.133	Nil
1	150	150/300 = 0.500	0.500
2	70	70/300 = 0.233	0.466
3	30	30/300 = 0.100	0.300
4	10	10/300= 0.033	0.132
Total	300	1.000	1.400

Step 2 :

Total no. of breakdowns per day = 1.40

Cost of breakdown per day = 1.40 × 650 = Rs. 910

Cost of preventive maintenance programme per day = Rs. 200+ Rs. 650 = Rs. 850

Expected annual savings from the preventive maintenance programme = (910 – 850) × 300 days

= 60×300 = Rs. 18,000

Answer for Q.NO.8.

Cost of machine, C = Rs. 15,000 + Rs. 3,500 = Rs. 18,500

Scrap value, S = Rs. 1,500.

Year	Maintenance Cost, M_1 (Rs.)	Cumulative Maintenance Cost, ΣM1 (Rs.)	Cost of Machine – Scrap Value (Rs.)	Total Cost T(n) (Rs.)	Annual Cost A(n) (Rs.)
(i)	**(ii)**	**(iii)**	**(iv)**	**(v)= (iii)+(iv)**	**(vi)=(v)/n**
1	260	260	17,000	17,260	17,260
2	760	1,020	17,000	18,020	9,010
3	1,100	2,120	17,000	19,120	6,373
4	1,600	3,720	17,000	20,720	5,180
5	2,200	5,920	17,000	22,920	4,584
6	3,000	8,920	17,000	25,920	4,320
7	4,100	13,020	17,000	30,020	4,288*
8	4,900	17,920	17,000	34,920	4,365
9	6,100	24,020	17,000	41,020	4,557

Lowest average cost is Rs.4,288 approx., which corresponds to n = 7 in above table. Thus machine needs to be replaced every 7th year.

Answer for Q.NO.9.

(a) Computation of failures & Mean life

Month (X)	Probability of Failure (P)	P X	
1	0.10	0.10	Average Life of a component = 3.35 Months
2	0.15	0.30	
3	0.25	0.75	
4	0.30	1.20	
5	0.20	1.00	
		Σpixi = 3.35 month	

Average No. of Replacements = 2000/3.35 = 597 per month

Cost of Individual Replacement = 597 × Rs. 3 = Rs. 1791 per month

Computation of expected No. of Replacements:

Month	Expected number of components to be replaced by the month end	
1	N1 = N0P1 = 2000 × 0.1	200
2	N2 = N0P2 + N1P1 = 2000 × 0.15 + 200 × 0.1	320
3	N3 = N0P3 + N1P2 + N2P1 = 2000 × 0.25 + 200 × 0.15 + 320 × 0.1	562
4	N4 = N0P4 + N1P3 + N2P2 + N3P1 = 2000 × 0.3 + 200 × 0.25 + 320 × 0.15 + 562 × 0.1	754.2

5	N5 = N0P5 + N1P4 + N2P3 + N3P2 + N4P1 = 2000 × 0.2 + 200 × 0.3 + 320 × 0.25 + 562 × 0.15 + 754.2 × 0.1	699.72

Computation of Average cost

Month (x)	Cumulative number of component Replace individually by month end	Cost		Total Cost (Tc)	Average
		Individual	Group		
		Rs	. Rs.	Rs.	Rs. per month
1	200	600	2000	2600	2600
2	520	1560	2000	3560	1780
3	1082	3246	2000	5246	**1748.67***
4	1836.2	5508.6	2000	7508.6	1877.15
5	2535.92	7607.76	2000	9607.76	1921.55

Since the average cost is lowest in 3rd month, the optimal interval i.e. replacement is 3 months. Also the average cost is less than Rs. 1791 of individual replacement, **the group replacement policy is better**.

(b) Let 'K' be the cost of Individual Replacement

Month	Average Cost of Group Replacement	Average cost of Individual Replacement	'K' Value* (Rs.)	
1	(2000 + 200 K)/1	597 K	5.04	* To obtain the value of K use the equation Average cost of Individual Replacement = Average Cost of Group Replacement
2	(2000 + 520 K)/2	597 K	2.97	
3	(2000 + 1082 K)/3	597 K	**2.82**	
4	(2000 + 1836.2 K)/4	597 K	3.62	
5	(2000 + 2535.92 K)/5	597 K	4.45	

If group replacement is anything smaller than 2.82, then Group Replacement would be uneconomical.

(c) Let 'a' be the unit cost of Group Replacement Policy

Month	Average Cost of Group Replacement	Average of Individual Replacement	'a' Value (Rs.)
1	(2000 a + 600)/1	1791	0.60
2	(2000 a + 1560)/2	1791	1.01
3	(2000 a + 3246)/3	1791	**1.06**
4	(2000 a + 5508.6)/4	1791	0.83
5	(2000 a + 7607.76)/5	1791	0.67

When unit cost is more than Rs. 1.06 then Individual Replacement policy would be better.

Answer for Q.NO.10.

Chart showing Optimal Replacement Period

Average life of the pole - 1 × 0.01 + 2 × 0.02 + 3 × 0.03 + 4 × 0.05 + 5 × 0.07 + 6 × 0.12 + 7 × 0.20 + 8 × 0.3 +

9 × 0.16 + 10 × 0.04 = 7.05 years.

No. of poles to be replaced every year = 5000 / 7.05 = 709

Average yearly cost on individual replacement = 709 × Rs.160 = Rs.1,13,440.

Group Replacement: Initial Cost = 5,000 × Rs.80 = Rs.4,00,000.

Year	No. of poles to be replaced	Yearly cost of individual replacement @ Rs. 160/pole (Rs.)	Cumulative cost of individual replacement (Rs.)	Total cost of individual replacement as well as group replacement (Rs.)	Average Annual Cost = Total Cost / Year (Rs.)
1	5,000 × 0.01 = 50	8,000	8,000	4,08,000	4,08,000
2	5,000 × 0.02 + 50 × .01 = 101	16,160	24,160	4,24,160	2,12,080
3	5,000 × 0.03 + 50 × 0.02 + 101 × 0.01 = 152	24,320	48,480	4,48,480	1,49,493
4	5,000 × 0.05 + 50 × 0.03 + 101 × 0.02 + 152 × 0.01 = 256	40,960	89,440	4,89,440	1,22,360
5	5,000 × 0.07 + 50 × 0.05 + 101 × 0.03 + 152 × 0.02 + 256 × 0.01 = 362	57,920	1,47,360	5,47,360	1,09,472
6	5,000 × 1.2 + 50 × 0.07 + 101 × 0.05 + 152 × 0.03 + 256 × 0.02 + 362 × 0.01 = 6023	9,63,680	11,11,040	15,11,040	2,51,840

Optimal replacement at the end of the 5th year.

Answer for Q.NO.11.

(i) Product B, with higher MTBF (i.e. 40 hours) than Product A (i.e. 30 hours), is more reliable since it has lesser chance of failure during servicing.

(ii) By MTTR we mean the time taken to repair a machine and put it into operation. Thus Product B, with lesser MTTR (i.e., 2 hours) than Product A (i.e., 5 hours), has greater maintainability.

(iii) Availability of a machine/product = MTBF / MTBF + MTTR

Therefore, Availability of Product A = 30/ (30+5) = 30/35 = 85.714% Availability of Product B = 40/(40+2) = 40/42 = 95.238%

Hence, Product B has more availability.

Answer for Q.NO.12.

First, let us compute the cost of a totally breakdown maintenance policy.

The expected number of months between failures

= 0.1 (1) + 0.2 (2) + 0.3 (3) + 0.4 (4) = 3.0

Cost per month of totally breakdown maintenance policy

= (No. of trucks) (Cost per breakdown) / (Expected number of months between failure)

= (50)(Rs. 3000) / (3.0) = Rs. 50,000

Now let us compute the costs of different periodicities of preventive maintenance.

(i) Preventive maintenance (PM) period one month

No. of breakdowns within the period of one month:

$B1 = (50) \times (0.1) = 5$

Cost of breakdown = 5 × Rs. 3000 = Rs. 15,000

Cost of preventive maintenance = Rs. 500 × 50 = Rs. 25,000

Total Cost during the PM period = Rs. 40,000

Therefore, cost per month for this policy is

= 40,000 ÷ 1 = Rs. 40,000

(ii) Preventive maintenance (PM) period two months

No. of breakdowns within 2 months:

$B2 = (50) \times (0.1 + 0.2) + (50) \times (0.1) \times (0.1)$	= 15.5
Cost of breakdown = (15.5) × Rs. 3000	= Rs. 46,500
Cost of prev. maintenance = Rs. 500 × 50	= Rs. 25,000
Total cost during the PM period	= Rs. 71,500

Therefore, cost per month for this policy:

Rs. 71,500 ÷ 2 months = Rs. 35,750

(iii) Preventive maintenance period 3 months

No. of breakdowns within 3 months:

$B3 = (50)\times (0.1 + 0.2 + 0.3) + (50 \times 0.1) (0.1 + 0.2) + (50 \times 0.1 \times 0.1) (0.1)$

$= 30 + 1.5 + 0.05 = 31.55$

Cost of breakdown = 31.55 × Rs. 3000	= Rs. 94,650
Cost of preventive maintenance = 50 × Rs. 500	= Rs. 25,000
Total	= Rs. 1,19,650

Therefore, cost per month for this policy

= Rs. 1,19,650 ÷ 3 months = Rs. 39,883.33

(iv) Preventive maintenance period 4 months

No. of breakdowns within 4 months

$B4 = [(50) \times (1.0)] + [(50) \times (0.1) \times (0.1 + 0.2 + 0.3) + (50\times 0.1 \times 0.1) \times (0.1 + 0.2) + (50 \times 0.1 \times 0.1 \times 0.1) \times (0.1) + (50 \times 0.1 \times 0.2) \times (0.1)]+ [(50 \times 0.2) \times (0.1 + 0.2) + (50 \times 0.2 \times 0.1) \times (0.1)] + [(50 \times 0.3 \times (0.1)]$

$= 57.855$

Cost of breakdown = (57.855) × (Rs. 3,000) = Rs. 1,73,565

Cost of preventive maintenance = 50 × Rs. 500 = Rs. 25,000

Total = Rs. 1,98,565

Therefore, cost per month for this policy is Rs. 1,98,565 ÷ 4 months = Rs. 49,641.25
Comparing the costs per month of different policies, we see that the policy of preventive maintenance every two months is the most economic policy.

THE END